2nd Physics exam - SCAN FIRST
November 19, 2021 ( 1 h 30 min )
Correction and Grading scale

## Exercise 1 : Optics - Lens doublet [3pts.]



## Exercise 2: Optics - Archeology of teaching accessories [6pts.]

1. For the mirror, the object and the image have the same size. Hence, the mirror does not change the magnification of the whole system.
all the rays are refected onto the mirror according to Snell-Descartes' law. In particular, as the mirror makes an angle of $45^{\circ}$ with the optical axis, a ray on the optical axis will be reflected at $90^{\circ}$ (figure needed to explain what happens on the mirror).
2. We expect here a ray-diagram with the object, the thin converging lens and the image.

All useful quantities should be defined on the ray-diagram : $\overline{A B}, \overline{A^{\prime} B^{\prime}}$, the center $O$ of the lens and its focal points $F$ and $F^{\prime}$.

We make the assumption that paraxial approximation can be used.
3. The image is reversed (converging lens with real object and real image).
4. $\gamma=\frac{\overline{O A^{\prime}}}{\overline{O A}}$ and $\frac{1}{\overline{A^{\prime} B^{\prime}}}-\frac{1}{\overline{A B}}=\frac{1}{f^{\prime}}$
we get $\overline{O A}=\left(\frac{1}{\gamma}-1\right) f^{\prime}$
numerical application : $\gamma=-3$ and $\overline{O A} \approx 423 \mathrm{~mm}$.
$\overline{O A}$ is compatible with the overhead projector specifications.
5. $\overline{O A^{\prime}}=(1-\gamma) f^{\prime}=1268 \mathrm{~mm}$.

The overhead projector has to be placed $\approx 1.3 \mathrm{~m}$ away from the screen. It is realistic.
discussion about the magnification

Clear explanation + scheme with at least one ray reflected onto the mirror

| clear ray-diagram | 10,5 |
| :--- | :---: |
| definition of all <br> quantities | 10,5 |


| assumption | $+0,5 B O N \not \subset S$ |
| :--- | ---: |
| answer + explana- <br> tion | $/ 0,5$ |
| 2 formulae | $/ 0,25$ <br> each <br> litteral expression <br> values with units <br> conclusion |
| literal expression + | $/ 0,5$ |
| value | $/ 0,5+/ 0,5$ |
| conclusion | $/ 0,5$ |

## Exercise 3 : Measures - friction due to viscous liquids. [5pts.]

| l. $f$ so $[f]=M L T^{-2}$ | Dimension of f | $/ 0,25$ |
| :--- | :--- | ---: |
| $[6 \pi \eta a v]=[6 \pi][\eta][a][\nu]=1 \times[\eta] \times L \times L T^{-1}$ | Dimension of the <br> other parameters <br>  <br> $=[\eta] \times L^{2} T^{-1}$ | $/ 0,5$ |
| therefore $[\eta]=\frac{M L T^{-2}}{L^{2} T^{-1}}=M L^{-1} T^{-1}$ |  | $/ 0,25$ |
| $2 . \eta$ is in. $\mathrm{cm}^{-1} \cdot \mathrm{~s}^{-1}$ |  | $/ 0,25$ |
| $3 . \eta=0,8400\left(10^{-3} \mathrm{~kg}\right) .\left(10^{-2} \mathrm{~m}\right)^{-1} \cdot \mathrm{~s}^{-1}=0,08400 \mathrm{~kg} \cdot \mathrm{~m}^{-1} \cdot \mathrm{~s}^{-1}$ | Dimension of $\eta$ | $/ 0,5$ |
| $4 .\left[v_{l i m}\right]=L T^{-1}$ |  | $/ 0,25$ |

\begin{tabular}{|c|c|c|}
\hline \[
\begin{aligned}
\& {\left[\rho_{s}-\rho_{m}\right]=[\rho]=M L^{-3}} \\
\& {[g]=L T^{-2}}
\end{aligned}
\] \& No + in the dimensional equation and correct expression of the dimension of \(\rho\) Dimension of \(g\) \& \(\begin{array}{r}10,5 \\ \\ \hline 0,25\end{array}\) \\
\hline \begin{tabular}{l}
\[
\begin{aligned}
{\left[\frac{2}{9} \frac{\left(\rho_{s}-\rho_{m}\right) g a^{2}}{\eta}\right] } \& =\frac{2}{9} \frac{\left[\rho_{s}-\rho_{m}\right][g][a]^{2}}{[\eta]} \\
\& =1 \times \frac{M L^{-3} \times L T^{-2} \times L^{2}}{M L^{-1} T^{-1}}=L T^{-1}
\end{aligned}
\] \\
Both terms have the same dimension, so the formula is thus homogeneous.
\end{tabular} \& \begin{tabular}{l}
Dimension of the other quantities and correct expression \\
Clear conclusion showing the relation between dimensional analysis and homogeneity of the formula
\end{tabular} \& /0,5

/0,25 <br>
\hline 5. $a=5,00.10^{-3} \mathrm{~m}, \rho_{s}=2,53.10^{3} \mathrm{~kg} \cdot \mathrm{~m}^{-3}$ et $\rho_{m}=0,91.10^{3} \mathrm{~kg} . \mathrm{m}^{-3}$ \& Correct conversion of $a$ $(/ 0,25), \rho_{s}$ et $\rho_{m}(/ 0,5)$, eventually of other quantities in the international system of units (or another coherent one). \& /0,75 <br>
\hline $v_{\text {lim }}=1,05 \mathrm{~m} . \mathrm{s}^{-1}$ \& Numerical application \& /0,25 <br>

\hline $$
v_{l i m}=3,78 \mathrm{~km} \cdot \mathrm{~h}^{-1}
$$ \& Correct conversion in $\mathrm{km} . \mathrm{h}^{-1}$ \& /0,5 <br>

\hline Comment : give all the points if the final result is correct, even without explanations. On the contrary, no unit implies no point. \& \& <br>
\hline
\end{tabular}

## Exercise 4 : Estimation of a battery discharging time [ $\approx 6$ pts. $]$



Figure 1 - Experimental setup

## Q1: 1.0 Pt

0.5 Pt
$U_{b}$ orientation and $+/-$ terminals
directed in the same way than $I$ (see Fig. 1).
The orientations of $U_{b}$ and $I$ then dictate the position of the $+/-$ terminals of the voltmeter and ammeter as shown in Fig.1.
(b) In the convention of Fig. 1: $U_{b}=e-r I$
2. Given the expression of $U_{b}$, the equation of the $I-V$ curve of the battery is :

$$
I=\frac{e-U_{b}}{r}
$$

$e$ can be deduced from the intercept of the $I-V$ curve while $r$ can be deduced from the inverse of the slope.
Considering the uncertainty boxes of Fig.2, the curves of minimum and maximum slope can be plotted. These curves are used to extract $e_{\min }$ (resp. $r_{\text {min }}$ ) and $e_{\text {max }}$ (resp. $r_{\text {max }}$ ).

Q2: 3.0 Pts
1 Pt
$I$ vs $U_{b}+$ extraction method of $e$ and $r$

1 Pt
plots of $I-V$ curves on Fig. 2 using uncertainty boxes


Figure 2 - Current-Voltage characteristics of the unknown battery

## Numerical values:

Note : In the following calculations, the most pessimistic case was systematically
considered by rounding up (resp. down) maximal (resp. minimal) values to the nearest graduation available on the graph: 0.2 V for voltages and 0.1 A for currents. Uncertainties will therefore be over-estimated since we did not considered rounding to the nearest half-graduations.

The extreme values of $e$ can be obtained from the intercepts of the 2 curves at $D \simeq(4.0 \mathrm{~V}, 0 \mathrm{~A})$ and $B \simeq(5.0 \mathrm{~V}, 0 \mathrm{~A})$ :

$$
e_{\min }=4.0 \mathrm{~V} \text { and } e_{\max }=5.0 \mathrm{~V}
$$

Therefore :

$$
\begin{aligned}
e & =\frac{1}{2}\left(e_{\max }+e_{\min }\right)=4.50 \mathrm{~V} \\
\Delta e & =\frac{1}{2}\left(e_{\max }-e_{\min }\right)=0.50 \mathrm{~V}
\end{aligned}
$$

Finally, keeping 2 significant digits, rounding $e$ to the closest $0.1 V$ and $\Delta e$ to the upper 0.1 V :

$$
e=(4.5 \pm 0.5) V
$$

Using the same methodology, the minimal slope $\left(s_{\min }\right)$ thus $r_{\text {max }}=s_{\min }^{-1}$ can be deduced from the 2 intercepts at $A \simeq(0 \mathrm{~V}, 2.3 A)$ and $B \simeq(5.0 \mathrm{~V}, 0 \mathrm{~A})$ :

$$
r_{\max }=\frac{1}{s_{\min }}=\frac{5.0 \mathrm{~V}}{2.3 \mathrm{~A}} \simeq 2.17 \Omega
$$

The maximal slope $\left(s_{\max }\right)$ thus $r_{\text {min }}=s_{\text {max }}^{-1}$ can be deduced from the 2 extreme points $C \simeq(0 V, 3.0 A)$ and $D \simeq(4.0 V, 0 A)$ :

$$
r_{\min }=\frac{1}{s_{\min }}=\frac{0.4 \mathrm{~V}}{3.0 \mathrm{~A}} \simeq 1.33 \Omega
$$

using $r_{\text {min }}$ and $r_{\text {max }}$ :

$$
\begin{gathered}
r=\frac{1}{2}\left(r_{\max }+r_{\min }\right) \simeq 1.75 \Omega \\
\Delta r=\frac{1}{2}\left(r_{\max }-r_{\min }\right) \simeq 0.43 \Omega
\end{gathered}
$$

Finally, keeping 2 significant digits by rounding $r$ to the nearest $0.1 \Omega$ and $\Delta r$ to the upper $0.1 \Omega$ :

$$
r=(1.8 \pm 0.5) \Omega
$$

3. (a) As we connect the battery to the load resistor, both device share the same voltage across their terminals with $I$ being the current flowing into the loop. The operating point therefore stands at the intersection point between the $I-V$ curve of the battery (in ASC) and the $I-V$ curve of the resistance (in PSC) given $U_{b}=R_{0} I$
Taking into account the uncertainties, two extreme operating points can be determined from Fig. 2 :
0.5 Pt
$e$ with uncertainties


$$
O P_{1} \simeq(3.2 \mathrm{~V} ; 0.6 \mathrm{~A}) \text { and } O P_{2}=(3.6 \mathrm{~V} ; 0.7 \mathrm{~A})
$$

(b) The power dissipated in the resistor writes : $P=R_{0} I^{2}$. From the 2 operating points we can determine the minimun (maximum) power :

$$
\begin{aligned}
& P_{\text {min }}=R_{0} I_{\text {min }}^{2}=(5 \Omega) \cdot(0.6 A)^{2}=1.80 \mathrm{~W} \\
& P_{\text {max }}=R_{0} I_{\text {max }}^{2}=(5 \Omega) \cdot(0.7 A)^{2}=2.45 \mathrm{~W}
\end{aligned}
$$

leading to :

$$
P=(2.1 \pm 0.4) W
$$

0.5 Pt

Power and uncertainties
4. From a dimensional analysis : $\left[C_{b}\right]=I \cdot T$. The discharge time $t_{d}$ is therefore homogenous to the ratio between the capacity $C_{b}$ and a current intensity. Taking the minimum and maximum operating current determined previously :

$$
\begin{aligned}
& t_{d}^{\min }=\frac{C_{b}}{I_{\max }^{o p}}=4.28 \mathrm{~h} \\
& t_{d}^{\max }=\frac{C_{b}}{I_{\min }^{o p}}=5.00 \mathrm{~h}
\end{aligned}
$$

leading to :

$$
t_{d}=(4.6 \pm 0.4) h
$$

The energy dissipated over the discharging time writes (with $t_{d}$ in seconds) : $E_{d}=P_{d} \cdot t_{d}$ The minimal/maximal enegy dissipated then write :

$$
E_{d}^{\min }=P_{d}^{\min } \cdot t_{d}^{\min } \text { and } E_{d}^{\max }=P_{d}^{\max } \cdot t_{d}^{\max }
$$

Using the previous values :

$$
E_{d}^{\min } \simeq 32.9 \mathrm{~kJ} \text { and } E_{d}^{\max }=39.8 \mathrm{~kJ}
$$

Finally :

$$
E_{d}=(36.3 \pm 3.4) k J
$$

