Physics test - Electromagnetism
June 20th 2022
Duration : 1h30
non programmable calculator allowed
no documents allowed

The marks will account not only for the results, but also for the justifications, and the way you analyze the results. Moreover, any result must be given in its literal form involving only the data given in the text. It is also reminded that the general clarity and cleanness of your paper may also be taken into account.

Exercise 1: Electrostatic accelerator ($\sim$ 7 pts)

An electrostatic accelerator is depicted in Figure 1-a. It consists of interdigitated thin metallic electrodes separated by a distance d, width W, deposited on an insulating substrate. A positive voltage bias $+V/2$ ($V > 0$) is applied to one series of K electrodes labelled P_i ($i \in 1, 2, \ldots, K$) while an opposite voltage bias ($-V/2$) is applied to the other ones, labelled N_i ($i \in 1, 2, \ldots, K$). The voltage difference between consecutive electrodes creates a piecewise **uniform electric field $\vec{E}$ along (Ox) axis, magnitude $E_0 = V/d$, within the free space between the electrodes. $\vec{E}$ remains null elsewhere (*i.e.* over the electrodes).**

In this exercise we will study the motion of a small metallic ball through the electrostatic accelerator. The ball will be considered as a point mass (m), thus only translational motion will be considered (no rotation) . The accelerator lies in the (xOy) plane and the position of the ball will be given by its coordinate $x(t)$ along (Ox) axis.

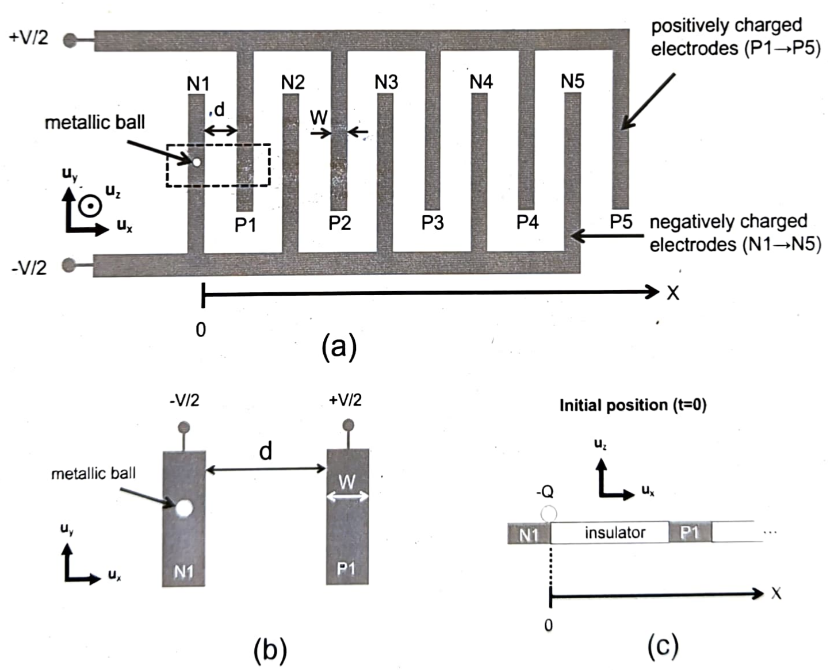

Figure 1: (a) General scheme of the electrostatic accelerator with $K = 5$ electrodes per branch. The dashed area is schematized in more detail in (b) and (c). (b) Magnified view of the dashed area in the (xOy) plane. (c) Cross sectional view in the (xOz) plane at initial time.

The acceleration principle is based on the **electrification** phenomenon. Given the metallic nature of the ball, **the latter gets instantly charged when placed into contact with an electrode**, carrying a **charge of the same sign as the voltage bias applied to the electrode**. More specifically, the ball holds a charge $+Q_0$ ($Q_0 > 0$) when rolling over a positively charged electrode (P_i) and a charge $-Q_0$ if the electrode is negatively charged (N_i). **When leaving an electrode, the ball keeps its polarization charge** (*i.e.* $\pm Q_0$), until coming into contact with the next electrode.

Results will be expressed in terms of W, d, Q_0, m and V.

Numerical values: $m = 2.0\,\mathrm{g}$, $d = 2.0\,\mathrm{cm}$, $W = 5.0\,\mathrm{mm}$, $Q_0 = 1.0\,\mu\mathrm{C}$, $V = 10.0\,\mathrm{V}$

1. Let $E(x)$ be the magnitude of the electric field along (Ox): $\vec{E}(x) = E(x)\,\vec{u}_x$ and $Q(x)$ the charge carried by the ball throughout the motion. Make a schematic plot of $E(x)$ and $Q(x)$ between N_1 and P_2.

As illustrated in Figure 1-b, the ball initially stands on the first electrode (N_1) thus holding a negative charge ($-Q_0$). The ball is then gently pushed across the boundary of N_1 at $x = 0$, falling under the influence of the electric field lying between N_1 and P_1 (Fig. 1-c). In the following, we study the motion of the ball along (Ox) denoting v_x the scalar value of its velocity along (Ox).

2. Name the force the ball is subjected to along (Ox) and plot its x-component between N_1 and P_2. Deduce and plot on the same graph the evolution of the logitudinal acceleration (dv_x/dt) justifying briefly your answer. Conclusion?

You can exploit the graph of Q.2 to answer the following questions:

3. Motion between N_1 and P_1
   The ball leaves N_1 without initial velocity: $v_x(t = 0) = 0$. Determine v_1, the velocity of the ball as it reaches P_1. Give the numerical value of v_1.

4. Motion over P_1
   Let τ be the time delay to cross P_1. Give τ in terms of W, d, Q_0 and V and give its numerical value.

5. Motion in subsequent regions
   Based on the responses to previous questions, give a qualitative description the motion of the ball after P_1. What is the total travelling distance L between N_1 and P_5 (*i.e.* from the right boundary of N_1 to the left boundary of P_5)? Over this path, what is the total length ℓ of the regions over which we observe a change in v_x?

6. Final velocity
   Find the final velocity of the ball (v_f) at P_5. Give its numerical value.

7. BONUS: What is the advantage of interdigitated electrodes instead of a single pair separated by a distance ℓ and subjected to the same voltage difference V?

Exercise 2: Magnetic field lines inside a torroidal coild (~ 3 pts)

A torroidal coil (Fig. 2-a) consists of a long insulated copper wire wound as joined spires along a ring-shaped frame with a square cross-section (Fig. 2-b). A current I is running in the wire hence creating a magnetic field $\vec{B}$ within the free volume of the torus. A simplified scheme showing the orientation of current lines in a cross-sectional plane of the coil is show in Fig. 2-c.

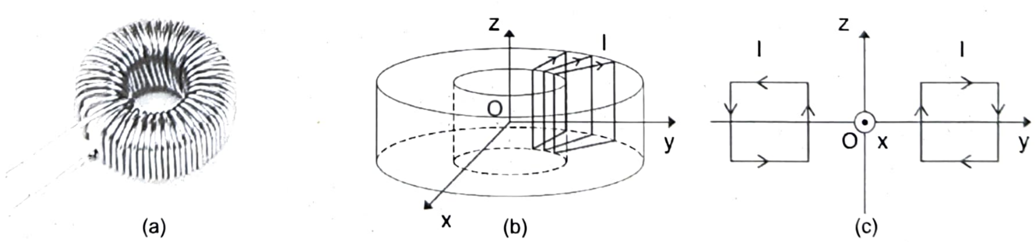

Figure 2: (a) Torroidal coil. (b) Schematic perspective view (only 3 consecutive spires are represented) and (c) cross-sectional view of the coil.

1. Analyze the symmetries and invariances of the electrical current distribution and deduce the **topography of the magnetic field** $\vec{B}$ created by current I **inside the coil** (*i.e.* within the spires).
   Note: Another frame than the cartesian frame may be used.

2. Determine the shape and the orientation of field lines inside the coil.

Exercise 3: Measuring the magnetic field created by a magnet ($\sim$ 10 pts)

Consider a very large U-shaped magnet in which we suspend a conducting frame $ABCD$ crossed by a positive constant current denoted I (generator not shown in Figure 3). I flows in the direction $A \to B \to C \to D \to A$ as denoted in Figure 3-a&b.

The frame has height $AB = CD = h$ and width $BC = DA = 2a$. We denote J the moment of inertia of the frame about the z axis. This frame is suspended by an insulating spring which only allows movement over two degrees of freedom: (i) vertical motion (Oz axis) and (ii) rotational motion (θ) around Oz axis. This spring has a stiffness k, a torsion constant C (creating a restoring torque: $-C\theta$). In addition, the frame undergoes a friction torque with respect to the Oz axis which is written $-\eta\dot{\theta}$. **Any induction phenomenon will be neglected here.**

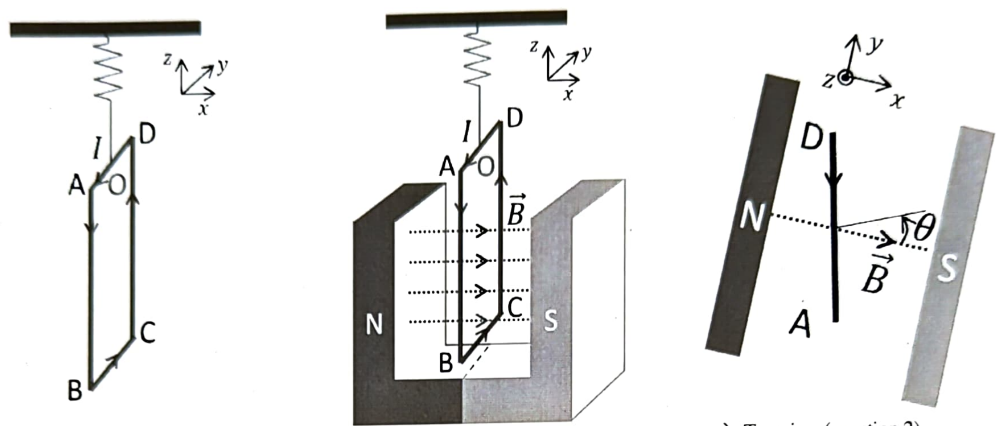

Figure 3: Frame with a current I running through it, partially immersed in a **B** field

In this exercise, it will be assumed that **segment AD is never subjected to the magnetic field created by the magnet, being outside of the U shaped magnet** and that only the two vertical segments AB, CD and the bottom horizontal segment BC are immersed in the magnetic field created by the magnet. We suppose the magnetic field to be constant and uniform over the surface of the frame with $\vec{B} = B_0\,\vec{u}_x$

Any result you provide should be expressed in terms of : k, η, C, B_0, h, a, I and J.

Numerical values: $g = 10\,\text{m}\,\text{s}^{-2}$, $k = 10\,\text{N}\,\text{m}^{-1}$, $h = 15.0\,\text{cm}$, $2a = 10.0\,\text{cm}$, $I = 2\,\text{A}$.

1. Study in static regime

1.1) Equilibrium of the frame without magnetic field (Figure 3-a)
Without the magnet (no magnetic field), the spring is elongated by $\Delta l_1 = 5.0\,\text{cm}$. Deduce the mass of the frame m detailing your analysis. Give the numerical value of m.

1.2) Equilibrium of the frame with a perpendicular magnetic field (Figure 3-b)
The frame is positioned inside the magnet such that $\theta = 0$. The elongation then becomes $\Delta l_2 = 7.0\,\text{cm}$. Deduce B_0 the magnitude of $\vec{B}$ field detailing your analysis. Give the numerical value of B_0.

2. Study in dynamic regime

The magnet is suddenly rotated such that the normal unit vector to the frame's surface and the magnetic field lines are making an initial angle $\theta(t=0) = \theta_0$ (Figure 3-c). Once released, the frame begins moving and returns back to an equilibrium position ($\theta = 0$) after a certain lapse of time.

2.1) Give the differential equation governing $\theta(t)$ then the general form of the solution assuming we observe small oscillations. Deduce the frequency of the free oscillations.

2.2) How much is the spring elongated after equilibrium?