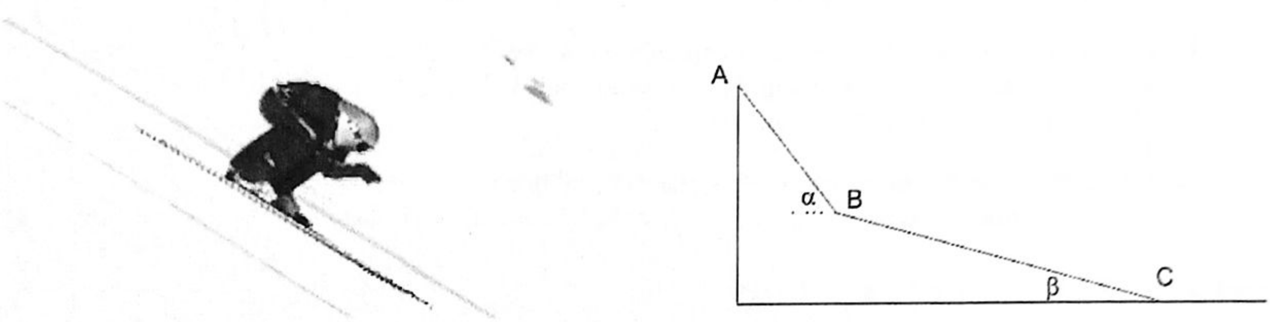
Physics exam 2 – Semester 2

June 2nd, 2023. Duration: 1h30

Not only your results, but especially your ability to justify them clearly and then analyze them critically will be evaluated. Furthermore, ***all results must be provided in a literal form using only the data from the statement.*** *It is also emphasized to pay attention to spelling and presentation of the answers. Any documents are prohibited.* ***Calculator is allowed.*** *The grading scale is provided as a reference.*

Exercise 1: Speed Skiing (~ 8 points)

The world speed skiing record was broken on Wednesday, March 22, 2023, in Vars (Hautes-Alpes), by the French skier Simon Billy, with a measured speed of 255.5 $\mathrm{km\,h^{-1}}$. The descent took place on the Chabrière slope, which is one of the two slopes approved by the International Ski Federation and authorized for speeds exceeding 200 $\mathrm{km\,h^{-1}}$.



(a) Speed skiing competitor

(b) Configuration of the Chabrière slope

Figure 1

We model the skier and its equipment (fig 1a) as a point of mass m. The coefficient of friction between the skis and the track is denoted as μ. The air resistance is modeled as a force whose magnitude varies proportionally with the velocity, with a proportionality coefficient denoted as k. The first portion of the track (part AB) is inclined at an angle α =45° with the horizontal axis (fig 1b), and air resistance can be neglected on this part of the track.

Questions 1 and 2 are independent.

1. Given that the skier arrives at B with a velocity v_B = 200 $\mathrm{km\,h^{-1}}$ after a time t_B = 6 s, determine the launch velocity v_A of the skier at A.
   Take m = 90 kg, g = 9.81 $\mathrm{m\,s^{-2}}$, μ = 0.015 for numerical calculations.

2. The second portion of the track is inclined at an angle β = 22.5° with respect to the horizontal axis (fig. 1b), and we consider air resistance with a value of the proportionality coefficient k = 3 $\mathrm{kg\,s^{-1}}$.

   (a) Write the differential equation satisfied by the magnitude of the velocity v.

   (b) Deduce the expression for the magnitude of the velocity as a function of time.

   (c) The skier's velocity was measured at point C with a value of 255.5 km/h. Calculate the numerical value of the skier's passage time t_C at C.

Exercise 2: Swing (~ 8 points)

We model a swing (fig. 2) as a homogeneous bar that is free to rotate around a fixed axis $(O'z)$, with a length L and a mass M. In equilibrium, the bar is in a horizontal position, and we denote J as the moment of inertia of the {bar + children} system with respect to $(O'z)$. The children are modeled as point masses A and B, each with a mass m, attached to the ends of the swing. The entire system {bar + children} then forms a single rigid body denoted as S, and its position is determined by the angle θ that the bar makes with respect to the horizontal. It should be noted that the angles are oriented with respect to the $(O'z)$ axis. The swing is subjected to the action of two identical springs with a spring constant k and a length at rest H. We consider the small angle approximation, so that the spring axes remain vertical and are at a distance D from the axis (OO'). We neglect any form of friction.

1. Provide an accurate assessment of the various mechanical actions exerted on S.
2. Show that the mechanical action of the two springs on the swing forms a torque, and provide the moment of this torque with respect to $(O'z)$ as a function of θ.
3. The system S is displaced from its equilibrium position without any initial velocity by a small angle θ_0 at time $t = 0$. Establish the equation of motion for the system.
4. Demonstrate that the general solution of this equation corresponds to oscillatory motion, and express the period of oscillation as a function of J, D, and k.
5. Give the expression for the moment of inertia J of the system S as a function of M, m, and L.

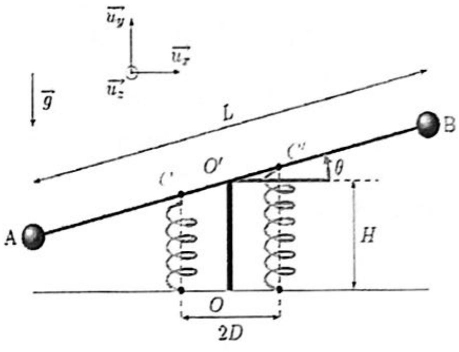


Figure 2

Exercise 3: The Gyrobus (~ 4 points)

The GYROBUS was used in the 1950s as an electric vehicle for public transportation, with the unique feature of not requiring electric power lines along the route for its electrical supply. The GYROBUS only needs charging stations installed regularly along the route to recharge its mechanical flywheel. The flywheel is a solid disk with a vertical axis housed in the bus floor (see figure 3).

During charging while the bus is stationary, the disk is rotated by an onboard electric motor, which is powered externally. When the bus is in motion, the opposite occurs: the rotation of the disk powers the electric motor, which in turn powers the driving wheels with a very high conversion efficiency assumed to be 100 % in the rest of the exercise.

The disk has a moment of inertia J with respect to its axis of rotation equal to $750\,\text{kg}\,\text{m}^2$. The bus (including the disk) is a vehicle with a total mass M of 10 tons.

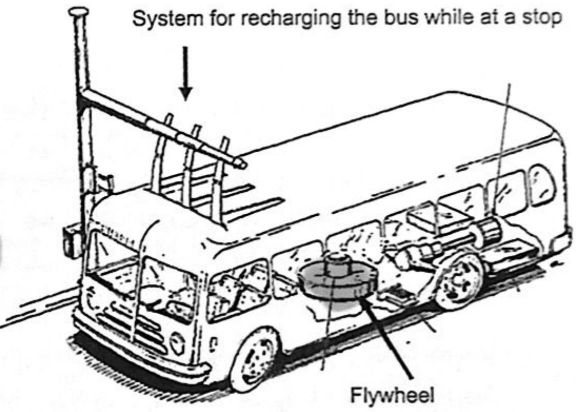


Figure 3

Additional information: When the bus is in motion, various frictional forces, considered independent of the speed, cause the bus to consume energy W_f equal to 0.4 kWh per kilometer traveled. The total kinetic energy of the gyrobus is equal to the sum of the translational kinetic energy of the vehicle and the rotational kinetic energy of the flywheel.

Question: The gyrobus is initially at rest and needs to climb a 2 km long hill with a vertical height of 150 m. What should be the minimum initial angular velocity of the flywheel for the gyrobus to reach the top of the hill with a speed of 50 km/h?