Semester 2 Final Physics Exam

June 21st 2023, Duration : 3 hours

Not only your results, but above all your ability to justify them clearly and analyze them critically will be accounted for. You are also reminded to take care with the spelling and presentation. Documents not allowed. Calculators allowed. The grading scale is only indicative.

Make sure you systematically give the **literal expressions** according to the exercise data.

Exercise 1 : Electricity – Position sensor (~ 7 pts ~ 1h05)

We propose to study an electrical sensor which is intended to measure changes in the position of a mobile element. The sensor consists in the association of different circuit elements which will be described progressively. To start with, the sensor features a sensing circuit depicted in Fig. 1. It consists in two immobile and identical coils (B1 and B2) wrapped around a U-shaped ferromagnetic material. In between the coils lies a mobile conductor (grey region in Fig. 1) which is located at the same distance δ from each coils at rest (see Fig. 1a).

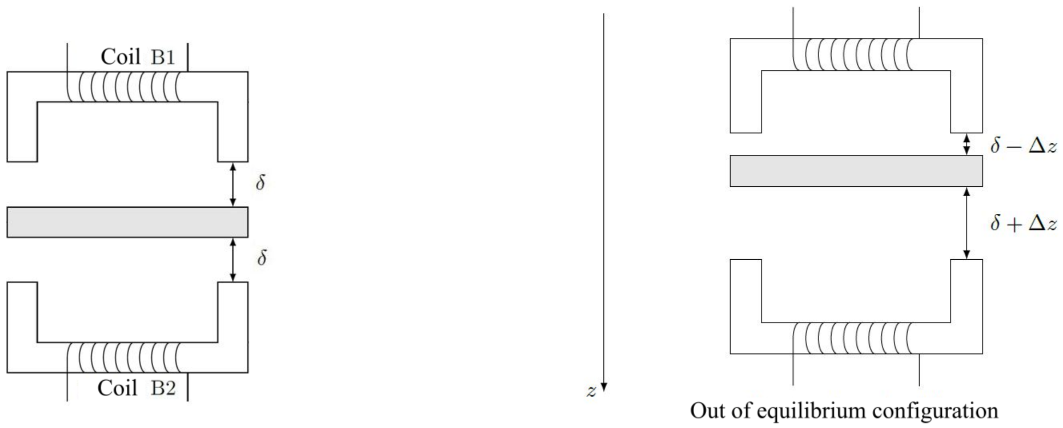
As the mobile element is moved by a distance Δz (see Fig. 1b) it induces a variation in the values of the coil's inductances L_1 and L_2 :

$$L_1 = L_0\left(1 - \frac{\Delta z}{\delta}\right) \text{ and } L_2 = L_0\left(1 + \frac{\Delta z}{\delta}\right)$$

where :

— L_0 is the nominal inductance of the coils when the mobile element is at rest (Δz=0, see Fig 1a)

— Δz is the algebraic displacement of the mobile conductor with respect to the rest position (Fig. 1b).



(a) initial configuration of the sensor, without displacement ($\Delta z = 0$)

(b) sensor configuration as the mobile element is moved by $\Delta z > 0$

FIGURE 1 – Description of sensing circuit

Coils B1 et B2 are fed by an AC voltage source, modeled as an ideal voltage source $e(t) = E\cos(\omega t)$, of angular frequency ω, in series with a resistance R (Fig. 2). R accounts for all the resistances in the circuit, including those of the coils.

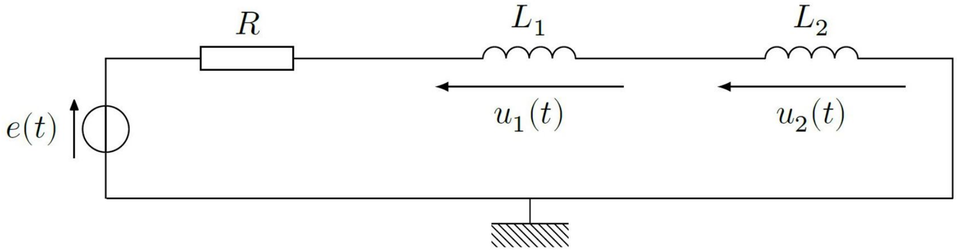


FIGURE 2 – Equivalent scheme of the sensing circuit

1. Establish the expression of $\underline{u}_1$ and $\underline{u}_2$ the complex voltages across B1 and B2 as defined in Fig. 2, in terms of R, L_1, L_2, ω and $\underline{e}$, the complex quantity associated to $e(t)$.

Then, $\underline{u}_1$ and $\underline{u}_2$ are used as input voltages of a substractor circuit, whose output voltage is $\underline{u}_s = \underline{u}_2 - \underline{u}_1$.

2. Show that the transfer function $\underline{H}(\omega) = \dfrac{\underline{u}_s}{\underline{e}}$ of the association {sensing circuit + substractor circuit} can be written in the following canonical form $\underline{H} = H_0 \dfrac{\jmath \dfrac{\omega}{\omega_0}}{1 + \jmath \dfrac{\omega}{\omega_0}}$, with $H_0 = \dfrac{\Delta z}{\delta}$ and ω_0 expressed in terms of L_0, R, Δz and δ.
3. Assuming $\Delta z > 0$: plot the Bode diagram of the <u>asymptotic gain</u> associated to $\underline{H}(\omega)$ in decibel. Provide the detailed calculations and relevant approximations used to obtain your plot.
4. Still with $\Delta z > 0$: what is the value of the phase-shift between the output and input as ω tends to 0 ? What if ω tends to infinity ? What if ω is equal to ω_0 ?
5. Still with $\Delta z > 0$: is $u_s(t)$ leading or lagging behind $e(t)$? Provide a brief justification.
6. How could we detect whether $\Delta z > 0$ or $\Delta z < 0$? Justify.
7. Determine the expression of the cutoff (angular) frequency ω_c at $-3\,dB$: justify.
8. How would you qualify this filter ? Enumerate all the useful associated characteristics of the filter.
9. In which frequency range should we use the filter to have $\underline{H}(\omega)$ independent of ω <u>and</u> proportional to the displacement Δz ?

We have $R = 750\,\Omega$, $L_0 = 60\,\text{mH}$ and we feed the sensing circuit with and AC voltage, frequency $f = 4\,\text{kHz}$.

10. Show that the output voltage can be written $u_s(t) = \dfrac{E\,\Delta z}{\delta} \cos(\omega t + \varphi)$. Specify the expression of φ.

We want the sensor to deliver a <u>DC</u> output proportional to the displacement Δz. To do so, we add to the association {sensing circuit + substractor circuit} a multiplier circuit to obtain the following output voltage : $s_m(t) = K_m\, e(t) \times u_s(t)$ (K_m being a positive constant).
It can be easily shown that $s_m(t)$ can be written as :

$$s_m(t) = K_m\, e(t) \times u_s(t) = K_m \frac{E^2\,\Delta z}{2\delta} \left[\cos(\varphi) + \cos(2\omega t + \varphi)\right]$$

.

Finally we add a filter after the multiplier circuit to achieve the final sensor (*i.e.* able to deliver the desired output). The final sensor thus features the following association : {sensing + substractor + multiplier + filter circuits}.

11. (a) What type of filter should we use ? Give a condition that must be satisfied by its cutoff frequency ω_{c2}.

(b) Give an example of such a filter obtained from a simple first order filter. Use equivalent Very Low and Very High Frequency (VLF, VHF) circuits to justify your answer.

For the mechanics section of this exam, we will consider that all the experiments are conducted in a galilean frame of reference.

Exercise 2 : Is it easy to maintain a pencil vertically in a state of equilibrium ? ($\sim$ 6 pts $\sim$ 55 min)

We are trying to determine whether it is easy to balance a pencil vertically on the tip of a horizontal finger, as shown in the figure 3.

FIGURE 3 – The pencil is balanced vertically on the tip of a finger.

Modeling

1. Make a free body diagram of the falling object (you are not asked to describe the fall itself here).
   Guidelines : i) we will consider that the vertical axis corresponds to a nil angle (e.g. $\theta = 0$), ii) the object does not slide at the point of contact with the finger and iii) falls without friction.

Nature of the Equilibrium

2. Explain why this represents a case of unstable equilibrium.

Minimally guided solution

An external factor (a slight movement, wind, etc.) can cause a small deviation from equilibrium. In this problem, we will consider that equilibrium is ruptured at initial time due to the presence of a small - but non zero - initial angular velocity.

Step 1 : Definition of the criterion to be used

We propose to study the time evolution of the angular velocity to see if it is more or less easy to restore balance.

3. Explain (without calculations) why this criterion may indeed be relevant.

Step 2 : Movement study

Given data :

— The moment of inertia J of a homogeneous bar (of length L, of masse M and small diameter) about an axis passing through its end is given by $J = \dfrac{1}{3}ML^2$.

— Second-order Taylor expansion around zero : $e^u \simeq 1 + u + \dfrac{1}{2}u^2$

4. Determine the expression for the time law of the angle at the beginning of the fall (small angles). Deduce the angular velocity in the very early moments of deviation from equilibrium : the approximation of the angular velocity at the beginning of the motion will be given as a polynomial of t of the 2^{nd} order at most.

Step 3 : Interpretation

5. Discuss the result obtained in question 4 and answer the initial question.

Exercise 3 : Using Laplace forces. (~ 7 pts ~ 1h)

We conduct the experiment depicted in figure 4. A horizontal rod, with a mass m, can slide without friction on two parallel horizontal rails, spaced apart by length a = AB. The rod is connected at its center J to a spring with a spring constant k. The origin O of the axis (Ox) corresponds to zero elongation of the spring. The rod allows the closure of an electrical circuit consisting of a constant electromotive force generator E and a resistance R, with R considered constant and equivalent to the series combination of all resistances in the circuit : the resistance of the rails, the internal resistance of the generator, and that of the rod. The entire setup is immersed in a vertical magnetic field $\vec{B} = B\vec{u}_z$, with $B > 0$.
Due to the presence of the magnetic field a Laplace force $\vec{F}_L$ is exerted on the rod :

$$\vec{F}_L = Ki(t)\,\vec{e}_x$$

where K is an algebraic constant - to be determined and justified from the given data - and $i(t)$ is the current intensity running in the circuit (defined positively from B to A).

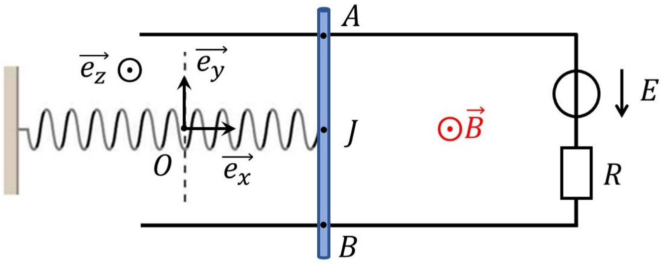


FIGURE 4 – An experiment involving Laplace rails.

1. The rod reaches its equilibrium position x_1. By providing a detailed explanation, find the expression of this equilibrium position.

We now turn off the voltage generator ($E = 0$) and the external magnetic field source ($B = 0$).

2. Determine the differential equation of motion for the rod and solve it.
3. Express $\mathscr{E}_{M0}$, the mechanical energy of the rod when E and B are zero, as a function of m, k, x and $\dot{x}$. Comment.

The same experiment is conducted again, keeping $E = 0$, but this time in the presence of the magnetic field $\vec{B}$. Due to the phenomenon of electromagnetic induction (which will be studied in the second year), even with $E = 0$ (a condition which remains satisfied throughout the rest of this exercise), a time-varying current $i(t)$ (defined positively from B to A) appears in the circuit during the motion of the rod.

4. Determine the new differential equation of motion for the rod without solving it.

$\mathscr{E}_M$ being the mechanical energy of the system, it can be shown that there is a power loss $\left(-\dfrac{d\mathscr{E}_M}{dt}\right)$ which is equal to the instantaneous power dissipated by Joule heating in the circuit.

5. Recall the expression for the instantaneous power dissipated by Joule heating in the circuit and show that $i(t)$ satisfies $i(t) = -\dfrac{aB\dot{x}}{R}$.
6. Rewrite the differential equation of motion for the rod, keeping the only unknown function $x(t)$.
7. As a function of m, a, k and R, provide the expression for the critical value B_c of the magnetic field, around which the dynamic behavior of the rod changes.
8. What is the nature of the solution $x(t)$ observed if $B < B_c$?
9. (Bonus question) Provide the expressions of the quantities that characterize this motion in terms of m, a, k, and R.