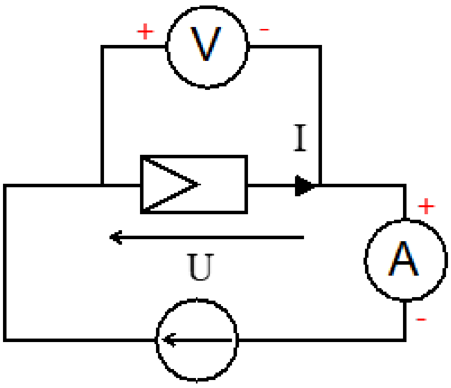
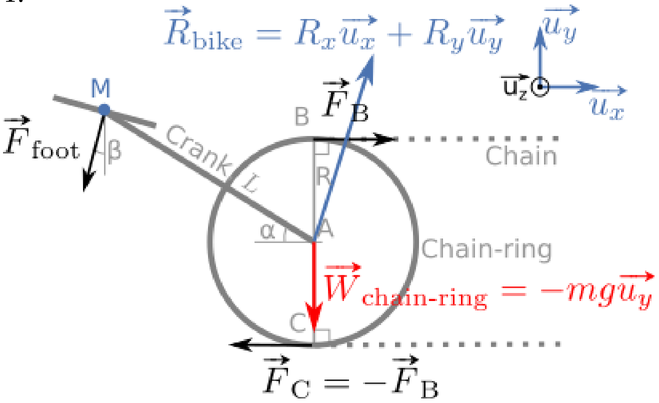
IE2 - Correction

Exercise 1 : Analysis of an I-V curve - 5 points

Elements of correction, expected items	grade
1. Given the chosen orientation for U and I we have a Passive Sign Convention (PSC)	0.5
2. 	1
3. In PSC the absorbed power writes : $P = U * I$ Reading I values on Fig.1 we get : • $U = 10\text{V}$, $I = -3\text{A} \Rightarrow P = -30\text{W}$ • $U = 20\text{V}$, $I \approx +3\text{A} \Rightarrow P = 60\text{W}$ In PSC, $P > 0$ means the PV modules absorbs power while it delivers power if $P < 0$ $\Rightarrow$ the module delivers power for $U = 10\text{V}$ and receives power for $U = 20\text{V}$.	1 (0.5pt for each power $\pm 5\text{W}$ can be accepted) 0.5
4.In PSC, the PV module delivers power as $P < 0$. Since $U > 0$ in Fig. 1 the module delivers power as $I < 0$. In the $[0V, 15V]$ region I is almost constant ($\approx -3\text{A}$) for increasing U values. Then I falls to zero from 15V to $\sim$ 19V. The maximum delivered power can thus be brought at $U = 15\text{V}$ $I = -3\text{A}$: $P_{\text{max}} \approx 45\text{W}$	1.5
5. From available data and a dimensional analysis : $P\text{lum} = A * G_m = 500\text{W}$ The efficiency is then : $\eta = P_{\text{max}}/P_{\text{lum}} \approx 9\%$	0.5

Exercise 2 : Static study of a bike transmission chain - 7 points

Elements of correction, expected items	grade
1.  *0.5 pts of the forces representation + 0.5 pts for the definition of the Cartesian basis and the forces expressions in that base (give points later if necessary).*	1
2. $\vec{\mathcal{M}}_{\vec{F}_{\text{foot}}}(A) = \vec{AM} \times \vec{F}_{\text{foot}}$ $= L\left(-\cos\alpha\vec{u_x} + \sin\alpha\vec{u_y}\right) \times \\|\vec{F}_{\text{foot}}\\| \left(-\sin\beta\vec{u_x} - \cos\beta\vec{u_y}\right)$ $= L\\|\vec{F}_{\text{foot}}\\| \left(\cos\alpha\cos\beta + \sin\alpha\sin\beta\right)\vec{u_z} = L\\|\vec{F}_{\text{foot}}\\|\cos(\alpha-\beta)\vec{u_z}$ *0.5 pts per vector + 1 pt for the result.*	2
3. The total moment resulting from the chain is $\vec{\mathcal{M}}_{\text{chain}}(A) = \vec{\mathcal{M}}_{\vec{F}_{\text{B}}}(A) + \vec{\mathcal{M}}_{\vec{F}_{\text{C}}}(A)$ $\boxed{\vec{\mathcal{M}}_{\text{chain}}(A) = -2R\\|\vec{F}_{\text{B}}\\|\vec{u_z}}$	1
4. At equilibrium, $\Sigma\vec{\mathcal{M}}_{\text{ext}}(A) = \vec{0}$ Since the weight $\vec{W}_{\text{chain-ring}}$ and the bike reaction force $\vec{R}_{\text{bike}}$ are both applied on A, their moments about point A are nil. As a result, at equilibrium we get $\\|\vec{F}_{\text{B}}\\| = \frac{L}{2R}\cos(\alpha-\beta)\\|\vec{F}_{\text{foot}}\\|$ $\boxed{\gamma = \frac{L}{2R}\cos(\alpha-\beta)}$	0.5 0.5 0.5
5. At equilibrium, $\Sigma\vec{F}_{\text{ext}} = \vec{0}$ $R_x = \\|\vec{F}_{\text{foot}}\\|\sin\beta$ $R_y = mg + \\|\vec{F}_{\text{foot}}\\|\cos\beta$	0.5 0.5 0.5

Exercise 3 : Kinematics - 8 points (+1.5pt bonus)

Elements of correction, expected items	grade
1. $\dim(a) = \dim(r) = \mathrm{L}$ $\dim(b)\, T^2 = 1$, hence $\dim(b) = \mathrm{T}^{-2}$	0.5
2. The radius r is constant, angle θ and altitude z vary with the same time dependency. Point M is thus moving on a cylinder, its motion is helical, of axis Oz and radius a. A full turn between times t_1 and t_2 corresponds a variation in angle θ of 2π : $\Delta\theta = \theta(t_2) - \theta(t_1) = 3b\left(t_2^2 - t_1^2\right) = 2\pi$. The change in altitude is then : $\Delta z = z(t_2) - z(t_1) = ab\left(t_2^2 - t_1^2\right)$ $\boxed{\Delta z = \frac{2a\pi}{3}}$ We can check that $\dim\left(\frac{2a\pi}{3}\right) = \mathrm{L}$. *One can note that $z = a\theta/3$ and use this directly.*	0.5 1.5
3. Position vector : $\overrightarrow{OM} = a\overrightarrow{e_r} + abt^2\overrightarrow{e_z}$ We differentiate with respect to time to get the velocity vector : $\overrightarrow{v} = \frac{\mathrm{d}\overrightarrow{OM}}{\mathrm{d}t} = a\dot{\theta}\overrightarrow{e_\theta} + 2abt\overrightarrow{e_r}$ $\boxed{\overrightarrow{v} = 6abt\overrightarrow{e_\theta} + 2abt\overrightarrow{e_z}}$ Note : we can check the homogeneity : $\dim(abt) = LT^{-2}T = LT^{-1}$ We differentiate again to get the acceleration vector : $\boxed{\overrightarrow{a} = \frac{\mathrm{d}\overrightarrow{v}}{\mathrm{d}t} = -36ab^2t^2\overrightarrow{e_r} + 6ab\overrightarrow{e_\theta} + 2ab\overrightarrow{e_z}}$ Note : we can check the homogeneity : $\dim(ab) = LT^{-2}$ and $\dim\left(ab^2t^2\right) = LT^{-4}T^2 = LT^{-2}$	0.5 1 1
4. The velocity in Frenet-Serret frame is $\overrightarrow{v} = \lVert\overrightarrow{v}\rVert\overrightarrow{u_T} = \frac{\mathrm{d}\ell}{\mathrm{d}t}\overrightarrow{u_T}$. We have thus $\overrightarrow{u_T} = \frac{\overrightarrow{v}}{\lVert\overrightarrow{v}\rVert} = \frac{2abt[3\overrightarrow{e_\theta}+\overrightarrow{e_z}]}{2abt\sqrt{9+1}}$ $\boxed{\overrightarrow{u_T} = \frac{3\overrightarrow{e_\theta}+\overrightarrow{e_z}}{\sqrt{10}}}$ *Remark : if $ab < 0$, the tangent vector is $\overrightarrow{u_T} = -\frac{3\overrightarrow{e_\theta}+\overrightarrow{e_z}}{\sqrt{10}}$*	0.5 1 *bonus 0.5*
5. This is the arclength at time t_1. Knowing that $\frac{\mathrm{d}\ell}{\mathrm{d}t} = \lVert\overrightarrow{v}\rVert = 2abt\sqrt{10}$, we get : $\ell(t_1) - \ell(0) = \int_0^{t_1} 2abt\sqrt{10}\,\mathrm{d}t$. Taking $\ell(0) = 0$, $\boxed{\ell(t_1) = ab\sqrt{10}\,t_1^2}$ Dimension of ℓ : $\dim\left(abt^2\right) = LT^{-2}T^2 = L$.	1.5
6. From the expression of $\overrightarrow{a}$ and $\overrightarrow{u_T}$, we see that $\overrightarrow{a} = 2ab\sqrt{10}\overrightarrow{u_T} - 36ab^2t^2\overrightarrow{e_r}$ In the Frenet-Serret frame, $\overrightarrow{a} = \frac{d\lVert\overrightarrow{v}\rVert}{dt}\overrightarrow{u_T} + \frac{\lVert\overrightarrow{v}\rVert^2}{R_c}\overrightarrow{u_N}$, with R_c the local radius of curvature. By identification, we find that $\overrightarrow{u_N} = -\overrightarrow{e_r}$ ($a > 0$, it is a radius), and $R_c = \frac{\lVert\overrightarrow{v}\rVert^2}{36ab^2t^2} = \frac{10}{9}a$	*bonus 1*