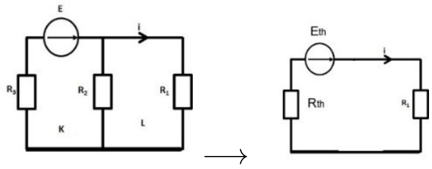
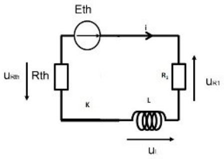
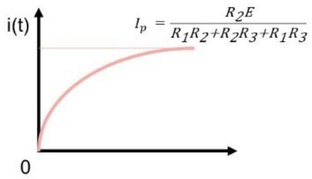
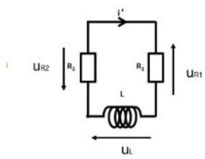
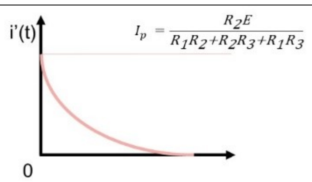
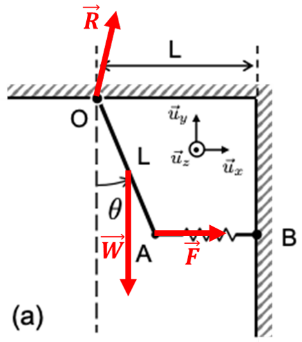
Correction of the SCAN 1st Physics exam – Semester 1
December 22nd, 2023

Exercise 1: Transient regime (10 points + 1 bonus)

1. In the steady state, the coil behaves like a closed switch  $E_{th} = \frac{R_2E}{R_2+R_3}$ and $R_{th} = \frac{R_2R_3}{R_2+R_3}$ Hence $I_0 = \frac{E_{th}}{R_1+R_{th}} = \frac{R_2E}{R_1R_2+R_2R_3+R_1R_3}$	0.5 (equivalent dipole) 0.5 (circuit simplification) 0.5 (I_0)
2a. Scheme with the different quantities:  (KVL) $E_{th} = (R_1 + R_{th})i + L\frac{di}{dt}$	1 (scheme) 0.5 (differential equation)
2b. Particular solution: $i_p(t) = \frac{E_{th}}{R_1+R_{th}} = I_0$ General solution of the complementary homogeneous equation: $i_h(t) = Ae^{-\frac{t}{\tau}}$ Initial condition: continuity of the current flowing through the coil $i(O^+) = i(0^-) = 0$ gives $A = I_0$ and $i(t) = I_0(1 - e^{-\frac{t}{\tau}})$	0.5 (i_p) 0.5 (i_h) 0.5 (initial condition) 0.5 ($i(t)$)
 2c.	0.5 +0.5 if τ shown on the scheme
2d. $i(t_\infty) = I_0$ This is logical since t_∞ represents the steady state	+0.5
 3a. (KVL) $u_L = (R_1 + R_2)i'$ with $u_L = -L\frac{di'}{dt}$ thus $L\frac{di'}{dt} + (R_1 + R_2)i' = 0$	0.5 (scheme) 0.5 (KVL) 0.5 (differential equation)
3b. general solution: $i'(t) = Be^{-\frac{t}{\tau'}}$ initial condition: $i'(O^+) = i'(0^-) = I$ gives $B = I$ and $i'(t) = Ie^{-\frac{t}{\tau'}}$	0.5 (general solution) 0.5 (expression i')
 3c.	0.25
3d. $W_{joule} = \int_0^\infty (R_1 + R_2)i'^2(t)dt = (R_1 + R_2)I^2 \int_0^\infty e^{-\frac{2t}{\tau'}}dt = \frac{1}{2}LI^2$	1.25
Energy stored in the coil at time $t = 0$: $W_0 = \frac{1}{2}LI^2$ Energy stored in the coil at time t_∞: $W_\infty = 0$ The difference corresponds to the energy dissipated in the resistances, in agreement with question 3d	0.5

Exercise 2: Statics ($\sim$ 10 points)

Part 1 - Study of the static equilibrium (6 points)

1. system = rod OA Referential : Earth (assumed to be Galilean) List of external forces : weight $\vec{W}$, force $\vec{F}$ exerted by the spring, reaction $\vec{R}$ of the wall  	1
2. $\vec{W} = -mg\vec{u_y}$ $\vec{F} = +k(L(1-sin\theta) - \ell_0)\vec{u_x}$ $\vec{R} = R_x\vec{u_x} + R_y\vec{u_y}$ At equilibrium, $\mathcal{M}_O(\vec{W}) + \mathcal{M}_O(\vec{F}) + \mathcal{M}_O(\vec{R}) = \vec{0}$ $\iff \overrightarrow{OG} \times \vec{W} + \overrightarrow{OA} \times \vec{F} + \vec{0} = \vec{0}$ Hence $-\frac{1}{2}mg.sin\theta + kL.cos\theta.(L(1-sin\theta) - \ell_0) = 0$ That is to say $mg.tan\theta + 2kL.sin\theta = 2k(L-\ell_0)$	1 (forces) *(0.5 if negative sign in the expression of $\vec{F}$)* 0.5 (expression of the moments around point O) 1 (final result)
3. Small angle approximation : $sin\theta \approx tan\theta \approx \theta$ therefore $mg.\theta_{eq} + 2kL\theta_{eq} \approx 2k(L-\ell_0)$ $\theta_{eq} \approx \frac{2k(L-\ell_0)}{mg+2kL}$ Numerical application: $\theta_{eq} \approx 0.415\ rad \approx 23.8°$	1
4. $\theta_{eq} = 0$ if $\ell_0 = L$ or if $k = 0$. This is expected for a regular pendulum.	0.5
5. We use $\vec{W} + \vec{F} + \vec{R} = \vec{0}$ we directly get $R_x = -k(L(1-sin(\theta) - \ell_0)$; $R_y = mg$ and $R_z = 0$	1

Part 2 - Rod crossed by a current I immersed in a $\vec{B}$ field (4 points)

1. Laplace force is exerted on the rod. It is equal to $\vec{F_\ell} = I.\vec{\ell} \times \vec{B}$ By considering it applies at the middle of the rod, we obtain $\vec{F_\ell} = -I.B_0.L(cos\theta.\vec{u_x} + sin\theta.\vec{u_y})$	1
The new equilibrium is obtained for $\mathcal{M}_O(\vec{W}) + \mathcal{M}_O(\vec{F}) + \mathcal{M}_O(\vec{R}) + \mathcal{M}_O(\vec{F_\ell}) = \vec{0}$ with $\mathcal{M}_O(\vec{F_\ell}) = -\frac{I.B_0.L^2}{2}.\vec{u_z}$ Hence at equilibrium, $-mg.sin\theta + kL.cos\theta.(L(1-sin\theta) - \ell_0) - \frac{I.B_0.L^2}{2} = 0$ We want $\theta = 0$ thus $B_0 = \frac{2.k.(L-\ell_0)}{I.L}$ Numerical application: $B_0 = 20\ T$	2.5
3. $B_0 = 0$ for $\ell_0 = L$. This is logical since we already have $\theta_{eq} = 0$ (see Part 1 question 4)	0.5