

Thermodynamics – IE n°2 - duration : 2 hours
No document allowed. All types of calculators allowed.
Both exercises are independent.

All the gases are supposed ideal $R = 8.31 \text{ J.K}^{-1}.\text{mol}^{-1}$

Exercise I : (~10pt)

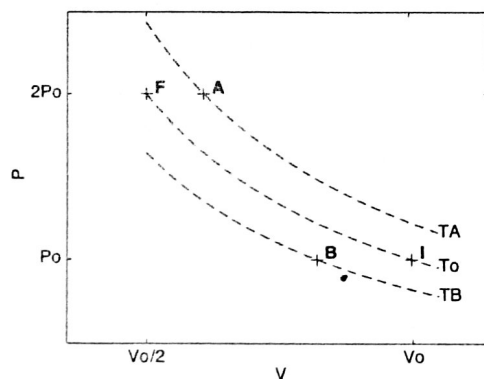
A first year INSA student prone of thermodynamics wishes to cut off his budget. Since he likes doing sport, he decides to develop a system to heat up his room while doing some sport. His idea is to convert the compression work of a gas to heat.

Both parts of this exercise are independent, but information given in the text in I-A can be useful for the I-B.

Given $P_0 = 1 \text{ atm} = 101325 \text{ Pa}$, $\overline{C}_v = \frac{R}{\gamma-1}$ $\overline{C}_p = \frac{R\gamma}{\gamma-1}$ taking $\gamma = 1.4$ for air.

I-A Model of the problem :

We consider a system of n_0 moles of air entrapped in a cylinder closed by a mobile piston. The system is disposed in a room and in the initial state (I), is at equilibrium with the air of the room which temperature and pressure are assumed constant and equal to T_0 and P_0 respectively. We write V_0 the volume of the system in the initial state. The idea is to compress the air contained in the cylinder until a state F and to release the piston so that the air is back to the initial state, therefore describing the following cycle :



Process 1 from I (T_0, P_0, V_0) to A ($T_A, P_A = 2P_0, V_A$): the air in the cylinder is first abruptly compressed by the student under a pressure of $2P_0$.

Process 2 from A to F ($P_F = 2P_0, T_F = T_0, V_F = V_0/2$): while maintaining a pressure at $2P_0$, the student waits until equilibrium is reached between the air in the cylinder and that of the room (assumed a thermostat at temperature T_0).

Process 3 from F to B ($T_B, P_B = P_0, V_B$): the student abruptly releases the piston.

Process 4 from B to I : the student waits until equilibrium is again reached.

- 1) Explain why the processes 1 and 3 can be assumed adiabatic and processes 2 and 4 are monobaric.
- 2) a) Give the expressions of the works transferred during the four processes W_1, W_2, W_3 and W_4 as a function of P_0, V_0, V_A and V_B .

- b) Give the expressions of the heat transferred during the four processes Q_1 , Q_2 , Q_3 , and Q_4 as a function of quantities to select within T_0 , T_A , T_B , \overline{C}_v , \overline{C}_p , n_0 , R .
- 3) Show that the heat received by the room during the cycle is equal to $P_0V_0/2$.
- 4) Writing by two different ways the work transferred during the compression (1 + 2), i.e. from I to F, show that $T_A = T_0 \frac{2\gamma-1}{\gamma}$. Similarly (thanks to the work of the expansion 3 + 4) show that $T_B = T_0 \frac{\gamma+1}{2\gamma}$.
- 5) Express the entropy variation of the system, then of the room during the cycle.
- 6) Check that this cycle is indeed possible but irreversible.

I-B Possibility of this project, orders of magnitude :

The volume of air in the room is $V = 25 \text{ m}^3$ and the pressure is constant, equal to P_0 . The room being fairly well insulated, when the student is back in the evening the temperature is equal to $T_{min} = 17^\circ\text{C}$. However, in order to work correctly, the student wishes a temperature equal to $T_{max} = 21^\circ\text{C}$. He makes a cylinder of length $L = 2.0 \text{ m}$ and cross section $S = 10 \text{ cm}^2$.

- 7) Calculate the number of moles of air n_{room} contained in the room.
- 8) Calculate the heat quantity Q_{total} required to increase the temperature of the room from T_{min} to T_{max} .
- 9) Assuming the compression is done under a pressure $2P_0$ (as in part A), what force must the student apply on the cross section of the cylinder in addition to the atmospheric pressure force? Do you think the value reasonable? (Justify).
- 10) How many cycles must the student operate to heat up his room? Do you find the result reasonable?

Exercise II : (~10pt)

A few years later, the same student, now working as an engineer, has to buy a water heater to produce hot water for his new home. He wants to use his thermodynamics 1st year expertise to choose the right technological solution and is considering the option of a condensing water heater.

A water heater being working continuously, in this exercise all quantities will be calculated per unit time (= per second) and will be written for instance \dot{n} for the number of moles per second, or \dot{Q} for the heat transfer per second.

The burner of the water heater is fed at constant pressure and temperature ($P_0 = 1.0 \text{ bar}$, $T_{g,i} = 25^\circ\text{C}$) by $\dot{n}_{\text{C}_3\text{H}_8,i} = 0.010 \text{ mol/s}$ of gaseous propane (C_3H_8) and dry air. The air introduced allows to have a quantity of dioxygen which is 1.4 time the stoichiometric proportion. The combustion of propane is total and leads to the formation of gaseous carbon dioxide and water. The gases coming out of the burner are evacuated to the chimney after having heated liquid water entering at $T_{w,l} = 15^\circ\text{C}$ and leaving at $T_{w,o} = 45^\circ\text{C}$. We write \dot{m}_{water} the mass flow (in g/s) of liquid water. The whole setup works at a constant pressure of 1.0 bar.

II-A 1st study :

The combustion gases (products + excess reactants) are evacuated into the chimney at temperature $T_{g,o} = 105^\circ\text{C}$; in these conditions the water formed by the combustion is completely in the gaseous state.

- 1) Complete the sketch of the setup on the result sheet, specifying temperatures.
- 2) Write the reaction of combustion of propane in the burner with formation of water vapor.
- 3) Complete the ICE table indicating the input, intermediate and output matter quantities (per second) on the result sheet (you'll call ξ the extent of reaction per second).
- 4) Calculate the molar standard reaction enthalpy for this reaction at 298 K ($\Delta_r H_{298}^0$).
- 5) a) Express the power \dot{Q}_r released by the reaction in these conditions of supply of the heater.
b) Using questions 1 and 3, express the power \dot{Q}_{water} necessary to heat up liquid water and the power \dot{Q}_{gas} necessary to heat up the combustion gases from $T_{g,i}$ to $T_{g,o}$.
- 6) Considering there is no thermal loss (adiabatic operation), establish a relation between \dot{Q}_r , \dot{Q}_{water} and \dot{Q}_{gas} .
- 7) Deduce the mass flow \dot{m}_{water} of hot water (quantity of water heated per second by the combustion gases).
- 8) What mass of propane must be used to give the water a total energy of 100 kWh ? What is the corresponding cost in euros?

II-A 2nd study :

The products of combustion are now evacuated at 45°C .

- 9) At 45°C , is the water formed by the combustion still completely in the vapour state? Justify your answer using the data below and the output amounts of the ICE table from question 3.
- 10) We call \dot{Q}'_r , \dot{Q}'_{water} and \dot{Q}'_{gas} respectively the power released by the reaction, the power necessary to heat up the liquid water and the power necessary to heat up the combustion gases in the cases where the products of combustion are evacuated at 45°C . Justifying your answer, indicate if \dot{Q}'_r , \dot{Q}'_{water} and \dot{Q}'_{gas} are lower, greater or equal to \dot{Q}_r , \dot{Q}_{water} and \dot{Q}_{gas} respectively. Deduce that the lowering of the output temperature for the combustion gases allows reducing the costs.

Data :

Thermodynamic properties of the gaseous compounds:

Compound	Molar mass (g/mol)	$\Delta_f H_{298}^0$ (kJ/mol)	\bar{C}_p (J. K ⁻¹ . mol ⁻¹)
C ₃ H ₈	44	-103.8	78.95
O ₂	32	0	28.97
N ₂	28	0	28.14
CO ₂	44	-393.5	39.80
H ₂ O _{vap}	18	-241.8	32.59

Molar heat capacity of liquid water : $\bar{C}(\text{liqu. water}) = 75.32 \text{ J. K}^{-1}. \text{ mol}^{-1}$

Saturating vapour pressure of water at 45°C : $P^*_{45^\circ\text{C}}(\text{water}) = 9583 \text{ Pa}$

Cost price of propane : 1.70 €/kg

Note : this exercise is mostly inspired from an exercise of the book « Thermodynamique – Applications aux systèmes physicochimiques » (Foussard *et al.* - Dunod, 2015)

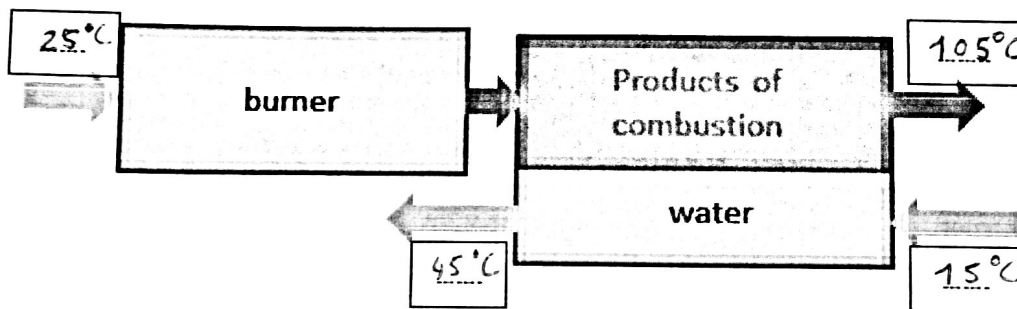
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Result sheet, to hand in.

Question II-A 1) : sketch of the setup, specify temperatures



Question II-A 3) : ICE table

Compound	C ₃ H ₈	O ₂	N ₂	CO ₂	H ₂ O	Total
Input of the burner (mol/s)	$\dot{n}_{C_3H_8,i} = 0,010$	$\dot{n}_{O_2,i} = 0,07$	$\dot{n}_{N_2,i} = 0,28$	$\dot{n}_{CO_2,i} = 0$	$\dot{n}_{H_2O,i} = 0$	$\dot{n}_{tot,i} = 0,36$
Intermediate state	$- \xi$	$- 5\xi$	0	$+ 3\xi$	$+ 4\xi$	$+ 1\xi$
Output of the burner (mol/s)	$\dot{n}_{C_3H_8,o} = 0$	$\dot{n}_{O_2,o} = 0,020$	$\dot{n}_{N_2,o} = 0,28$	$\dot{n}_{CO_2,o} = 0,030$	$\dot{n}_{H_2O,o} = 0,040$	$\dot{n}_{tot,o} = 0,37$

$\xi = 0,010$