Thermodynamics **Test May 17th 2024**

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⟵ Please enter you student number, and write your name above.

NAME, First Name :

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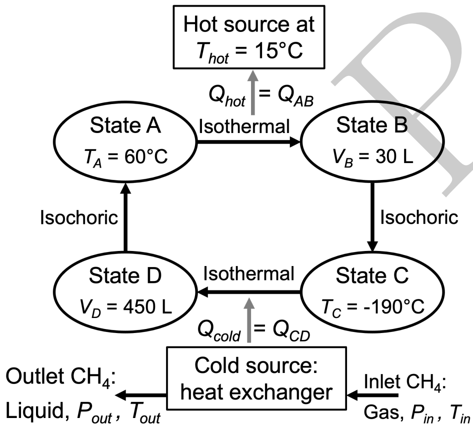
Duration : 2 h - All booklets, notes and calculators authorised - No computer or tablet, no wifi no 4/5G

Methane (CH_4) : thermodynamic data

Molar mass : $M_{CH_4} = 16\,g/mol$
Gaseous molar specific heat at constant volume : $\bar{C}_{V_m(CH_4)} = 27.4\,J/(mol\ K)$
Gaseous molar specific heat at constant pressure : $\bar{C}_{P_m(CH_4)} = 35.8\,J/(mol\ K)$
Latent heat at $T_{vap} = -162\,°C$: $L_{vap(CH_4)} = 511\,kJ/kg$
Saturating vapor pressure at $T_{vap} = -162\,°C$: $P^*_{CH_4} = 1\,bar$

1 The Stirling cryogenerator (15 points)



The Stirling cryogenerator is a refrigerator device allowing the obtention of liquid methane. It is made by a closed piston-cylinder that does not exchange shaft work and it contains an amount of moles of helium (He, an **ideal monoatomic gas**) equal to $n_{He} = 265\,mol$. Its working principle is based on the Stirling cycle made of the four reversible processes represented in the side figure. By using the data available in the exercise and the tutorial booklet, and by clearly explicating your reasoning :

Question 1 What is the numerical value of P_B (in bar) ?

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Question 2 Give and demonstrate the literal expression of the heat Q_{hot} exchanged in the process $A \to B$ as function of the available data.

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In isothermal processes for ideal gases in closed systems $\Delta_t U = 0$, so $Q_{AB} = -W_{pf(AB)}$ for the reversible process AB.
The work $W_{pf(AB)}$ is obtained from its definition $W_{pf(AB)} = -\int_A^B PdV = -\int_{V_A}^{V_B} \frac{n_{\text{He}} RT_A}{V} dV = -n_{\text{He}} RT_A \ln\left(\frac{V_B}{V_A}\right)$ with $V_A = V_D$ leading to : $Q_{hot} = Q_{AB} = n_{\text{He}} RT_A \ln\left(\frac{V_B}{V_D}\right)$ (2 pts).

Question 3 What is the numerical value of Q_{hot} in MJ ?

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Question 4 Give and demonstrate the literal expression of the heat Q_{BC} exchanged in the process $B \to C$ as function of the available data.

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In isochoric processes $W_{pf} = 0$ because $\Delta V = 0$. Moreover, being also $W_s = 0$ for this system, we have $\Delta_t U_{BC} = Q_{BC}$. Then, for ideal gases, $\Delta_t U = n\bar{C}_{V_m} \Delta_t T$ for any process. So, knowing that $T_B = T_A$, we have for the process BC : $Q_{BC} = n_{\text{He}} \bar{C}_{V_m(\text{He})}(T_C - T_A)$ (2 pts).

Question 5 What is the numerical value of Q_{BC} in MJ ?

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Question 6 Give the literal expression of the CoP as function of the available data.

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For refrigerators : $CoP = \frac{Q_{cold}}{W_{pf(tot)}} = \frac{Q_{CD}}{W_{pf(AB)} + W_{pf(CD)}}$ being $Q_{cold} = Q_{CD}$, $W_{pf(BC)} = 0$ and $W_{pf(DA)} = 0$. Then, $Q_{CD} = -W_{pf(CD)}$ and $W_{pf(CD)} = -n_{\text{He}} RT_C \ln\left(\frac{V_D}{V_B}\right)$ (isothermal with $V_C = V_B$), leading to : $CoP = \frac{-T_C \ln(V_D/V_B)}{T_A \ln(V_B/V_D) + T_C \ln(V_D/V_B)}$ (2 pts).

Question 7 Give the numerical value of CoP (attention : $CoP_{\text{ref}} \geq 0$, not 1).

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Let's focus now on the cold source. It is a heat exchanger (open system) where heat is obtained by a stream of methane $\dot{m}_{\text{CH}_4}$ =34.5 kg/s at $P_{in} = P_{out} = 1$ bar entering the exchanger in the gaseous state at T_{in} and leaving it as saturated liquid[1] at $T_{out} = -162$ °C.

Question 8 Considering that the rate of heat lost by the methane stream is $\dot{Q}_{\text{CH}_4}$ =−25 MW, give and demonstrate the literal expression of T_{in} as function of the available data.

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In the heat exchanger ($\dot{W}_{s(\text{CH}_4)} = 0$) $\Delta_s \dot{H}_{\text{CH}_4} = \dot{Q}_{\text{CH}_4}$ and $\Delta_s \dot{H}_{\text{CH}_4} = \frac{\dot{m}_{\text{CH}_4}}{M_{\text{CH}_4}} \bar{C}_{Pm(\text{CH}_4)}(T_{out} - T_{in}) + \dot{m}_{\text{CH}_4} L_{\text{cond}(\text{CH}_4)}$ with $L_{\text{cond}(\text{CH}_4)} = -L_{\text{vap}(\text{CH}_4)}$ due to the change of T and phase of the methane stream between inlet and outlet. So, $T_{in} = T_{out} - \frac{\dot{Q}_{\text{CH}_4} + \dot{m}_{\text{CH}_4} L_{\text{vap}(\text{CH}_4)}}{\dot{m}_{\text{CH}_4} \bar{C}_{Pm(\text{CH}_4)} / M_{\text{CH}_4}}$ (2 pts).

Question 9 What is the numerical value of T_{in} in °C ?

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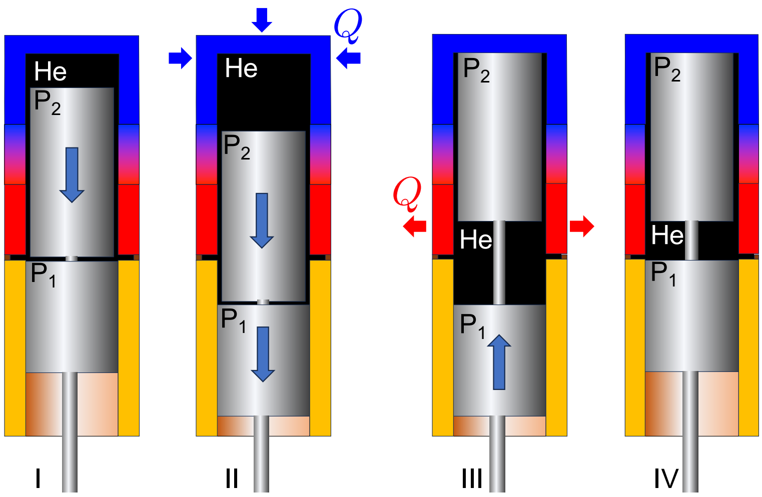
The Stirling Cycle in practice

A Striling cryogenerator is made of two pistons moving independently (piston "P_1" and "P_2"). The working gas (He) lies in a chamber represented in black on the figure. He gas can circulate around P_2 but not P_1. On the figure, are represented the four stages of the Stirling cooling cycle, but **the order is lost**.

Can you re-order them and tell which scheme corresponds to which state ?



Question 10 State A :

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Question 11 State B :

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Question 12 State C :

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Question 13 State D :

■ II ☐ I ☐ III ☐ IV

1. Saturated liquid : a liquid that is about to vaporise (vapour and liquid are at equilibrium).

2 Water lost during breathing (5 pts)

Question 14 How much water is lost by human body due to breathing in dry air during a day (give the result as a mass proportion of beverage water intake) ?
Indications : The average tidal volume of human lungs (volume of air moved into or out of the lungs in 1 breath) is 0.5 L. The average beverage water intake is 1.5 L per day.

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Own the problem : We have to calculate the proportion p of water lost due to breathing in one day related to the beverage water intake. We **are given** (undefined variables) the tidal volume : $V_t = 0.5\,\mathrm{L}$, the beverage water intake $V_b = 1.5\,\mathrm{L/day}$.
We **measure** the rate of breath : $r_b = (15 \pm 1)\,\mathrm{min}^{-1}$.
We **assume** the temperature of air moved out of lungs is $T = 37\,^{\circ}\mathrm{C}$, the vapor pressure of water in air at 37 °C is $P_w^* = 6.3\,\mathrm{kPa}$.
We **define** the number of mole of water expelled in one day : n_w, the molar mass of water : $M_w = 18 \times 10^{-3}\,\mathrm{kg/mol}$, the density of water : $\rho_w = 1000\,\mathrm{kg/m^3}$ (1 pts).
Resolution strategy : We assume that the air moved into the lungs is dry and that the relative humidity of air moving out of the lungs is 100%. We calculate the equivalent volume of liquid water transforming the water quantity into volume of liquid (1 pts).
Resolution : The proportion p is given by : $p = \frac{M_w n_w}{\rho_w V_b}$. Assuming ideal gas law $P_w^* V_t r_b = n_w R T$ (0.5 pts), we have :
$p = \frac{M_w P_w^* V_t r_b}{R T \rho_w V_b}$ (1 pts)
N.A. $p = 32\%$ (0.5 pts)
Assuming a 5% error in reading the water vapor pressure, 1 digit error in given data (20% and 7%) and 7% error in measuring r_b, we finally find : p=(32 ± 14) % (0.5 pts).
Critical look : The volume of water lost during breathing is between one third and one half of the beverage water intake. It is not most of it, but far from being negligible. Note that nose breathing decreases this proportion because water condensate on (cold) nose (0.5 pts).