INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON	*FIMI Departement* *SCAN 2nd*	*LAST NAME*	*FIRST NAME*

Mechanics of Systems – Semester 1 final test

Thursday 01/02/2024 – Duration 2h (10h-12h)

Kinematical analysis of two mixers

Authorised documents: personal formula sheets (2 A4 pages); calculator; 1 sheet on classic joints.

1 3D MIXER : ANALYTICAL ANALYSIS (~13 MARKS)

The mixer considered in this first part can be used in food-processing industry but also in chemical and construction industries.

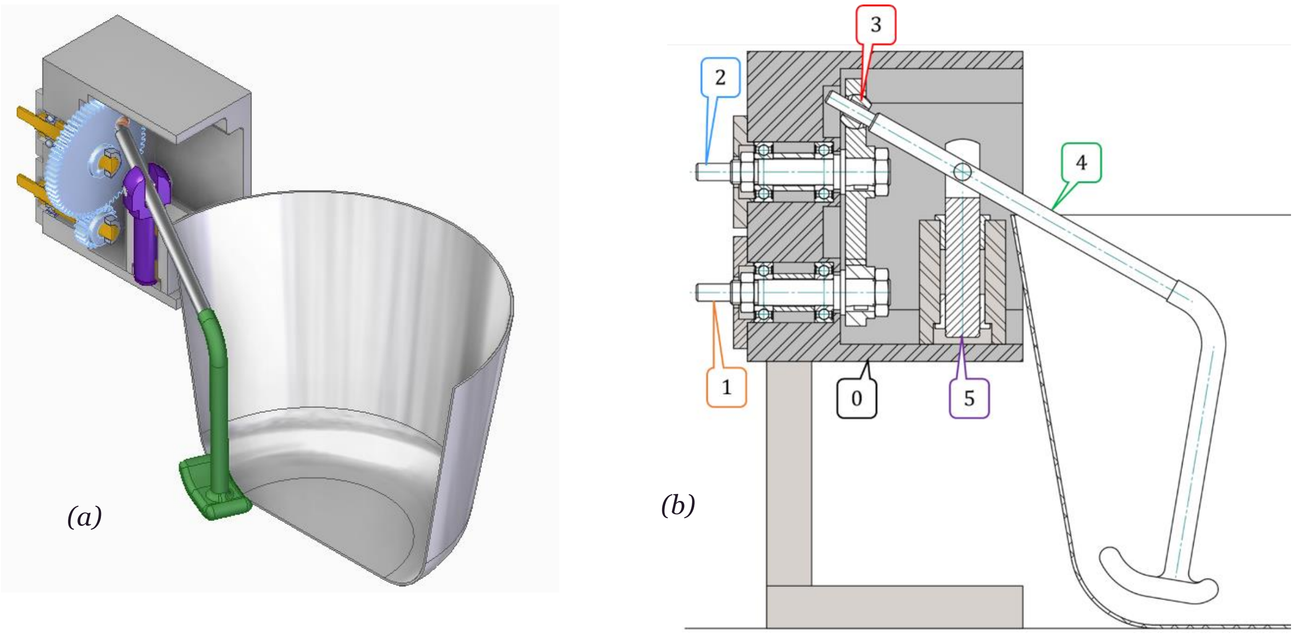


Figure 1. (a) CAD model of the mixer *(b) Technical drawing (cross-section) of the mixer*

The mechanism shown in Figure 1 and whose kinematic model is in Figure 2 (Annex 1) comprises 5 rigid solids:

- pinion S_1 connected to the ground S_0 by a revolute joint of axis $(A, \overrightarrow{x_{0,1}})$.
  1/0 motion parameter: $\alpha_1 = (\overrightarrow{y_0}, \overrightarrow{y_1})$.
- gear S_2 :
  - connected to the ground S_0 by a revolute joint of axis $(C, \overrightarrow{x_{0,2}})$,
    2/0 motion parameter: $\alpha_2 = (\overrightarrow{y_0}, \overrightarrow{y_2})$.
  - and in contact at point B with pinion S_1: the meshing condition between the mating teeth will be represented by a no-slipping conditions at point B.
    No parameter is associated with this joint.
- spherical solid S_3 connected to gear S_2 by a spherical joint,
  No parameter is associated with this joint.
- shaft S_5 connected to the ground S_0 by a revolute joint of axis $(E, \overrightarrow{z_{0,5}})$.
  5/0 motion parameter : $\theta_5 = (\overrightarrow{x_0}, \overrightarrow{x_5})$.
- spatula S_4 of centre of mass G_4 and extremity M:
  - connected to solid S_3 by a prismatic joint of direction $\overrightarrow{x_{3,4}}$.
    4/3 motion parameter : x such that $\overrightarrow{DE} = x\overrightarrow{x_{3,4}}$
  - and connected to shaft S_5 by a revolute joint of axis $(E, \overrightarrow{y_{5,4}})$.
    4/5 motion parameter : $\beta_4 = (\overrightarrow{z_5}, \overrightarrow{z_4})$.

The geometrical data are given in figure 2. Pinion S_1, operated by a motor (not shown here), is the input of the mechanism whereas the output corresponds to the three-dimensional mixing motion of spatula S_4.

1.1 PRELIMINARY STUDY

1. Draw the graph of links and change of basis diagrams using **the box in annex 1**.
2. Write and develop the rolling with no slipping condition at point B.

3. Express and develop the constraint equation(s) between S_2 and S_3 (project in the coordinate system 0).

4. What is the degree of mobility of the system? Justify.

1.2 KINEMATICAL SCREWS

Having determined the relationships between the various kinematical parameters in the preliminary study, this section focuses on the analysis of the various motions in the system.

5. Give the nature of motion 1/0 and deduce the nature of the corresponding kinematical screw (or wrench). Express its vector coordinates (sum and moment) at point A.

6. Express the kinematical screw 2/0 at point D. What is the trajectory of point D in its motion with respect to 0?

7. Express the kinematical screw 2/1 at point B. Determine the associated pitching and rolling vectors.

8. Give the nature of motion 4/3 and deduce the nature of the corresponding kinematical screw (or wrench). Express its vector coordinates (sum and moment) at point E.

Spatula S_4 is the output member of the mechanism and its motion 4/0 controls the mixing motion and characteristics.

9. Expression of the kinematical screw for motion 4/0 :
   a. Express its vector coordinates at point E.

   b. Determine the velocity with respect to the ground 0 of point M, tip of the spatula, in terms of geometrical data, parameters β, θ and their time-derivatives. The result should be express in the coordinate system attached to solid 4.

c. With no further calculations, give the velocity at point G_4 with respect to coordinate system O.

1.3 Analysis for pre-dimensioning

The mixer dimensions depend on the choice of motor and its operating regime, the selection of bearings for the revolute joint between S_0 and S_5 along with the knowledge of the loads applied on spatula 4. The motor drives pinion S_1 whose rotational speed is selected based on the desired mixing speed at point M.

In order to choose the proper bearings between S_0 and S_5, the corresponding angular speed needs to be known. From the constraint equations developed in question 3, the following relationship can be established:

$$\tan\theta = \frac{b}{a}\sin\alpha_2$$

10. Express the angular velocity vector $\overrightarrow{\Omega_{5/0}}$ in terms of the input parameter, its time-derivative and geometrical parameters.
NB1 : Use the following hypothesis on parameters : $\alpha_2 = 0$ *when* $\alpha_1 = 0$
NB2 : Reminder : $\frac{d}{dx}(\tan x) = 1 + \tan^2 x$

In order to analyse the actual loads on the spatula, the fundamental principle of Dynamics needs to be used (Newton's 2nd law- to be introduced in Mechanics of Systems during semester 4), which relies on the acceleration vector at the centre of mass G_4.

11. Calculate the acceleration vector with respect to the ground 0 at point G_4 in terms of geometrical data, parameters β, θ and their time-derivatives. In view of the geometry $f \ll e$, and in order to simplify the analytical expressions, **neglect the contributions of f ($f \approx 0$)**

2 2D MIXER: GRAPHICAL ANALYSIS (~7 MARKS)

In this second part, another technology of mixer for food processing is considered (see the figure beside) whose components all move in one plane. The joints between 0 and 2 at B, between 2 and 1 at C, between 1 and 4 at D, between 3 and 4 at E, and between 0 and 3 at F are perfect revolute joints, whereas solids 2 and 3 are two gears assimilated to two discs rolling without slipping at point I.

The kinematical model of the mixer is shown in figure 3 (Annex 2). A motor-reducer operates solid S_3 **clockwise** at a rotational speed $\boldsymbol{N_{3/0} = 4,5\ rev/min}$. In order to reproduce manual mixing conditions, the mixing speed should remain below $\boldsymbol{200\ mm/s}$. Moreover, the mechanism is designed such that the motions of the parts are **symmetrical** with respect to plane $(D, \vec{y}, \vec{z})$.

a) Which hypothesis ensures motion symmetry? If it exists, specify the instant centre of rotation for motion 2/3.

b) $\overrightarrow{V(E/0)}$ is given (see figure 3), justify the direction and sense in figure 3. Knowing that radius $FE = 70\ mm$, calculate its amplitude.

For each of the following questions, graphical constructions **should be made in figure 3** and the associated explanations will be provided in the **appropriate boxes below**.

c) Using the system specificity, draw $\overrightarrow{V(C/0)}$.

d) Using the system specificity, define the trajectory of point D in its motion with respect to 0. Draw the velocity vector $\overrightarrow{V(D/0)}$.

e) Determine graphically the instant centre for motion 4/0. Draw $\overrightarrow{V(G/0)}$.

f) Determine graphically the instant centre for motion 1/0.

g) Based on the result of the previous question, draw the velocity vector $\overrightarrow{V(A/0)}$.

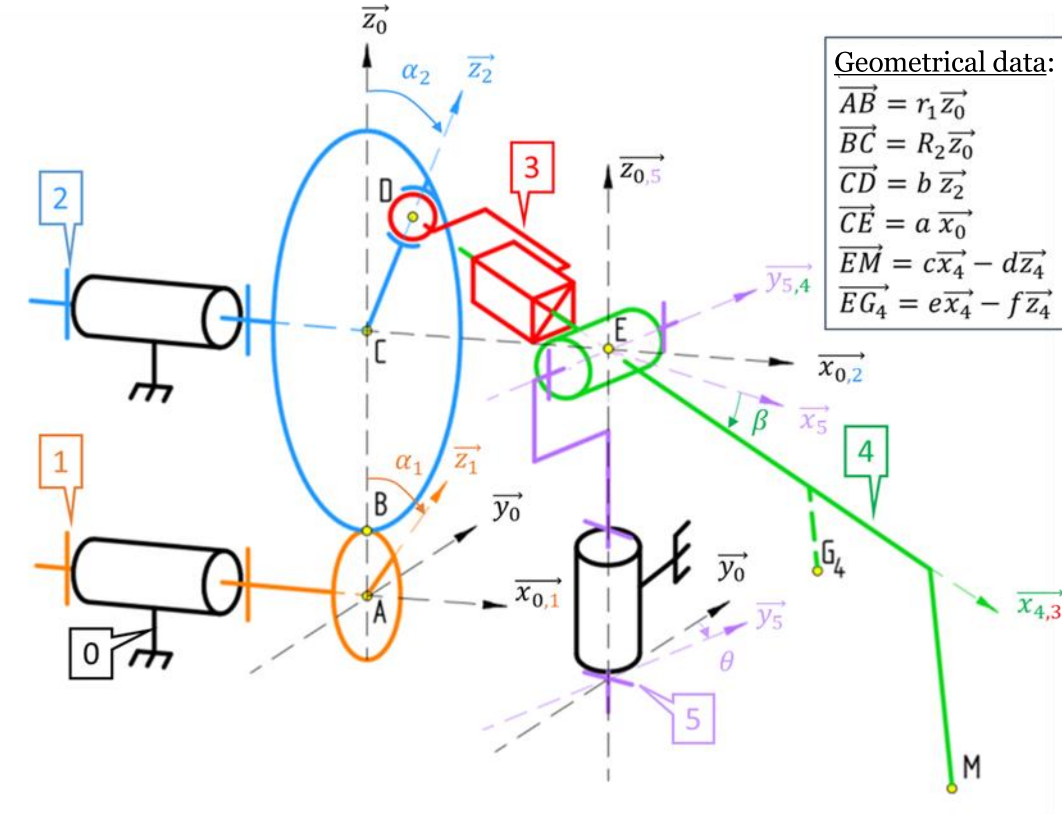
The mixing speed is defined as the horizontal velocity difference between points A and G as:

$$\overrightarrow{\boldsymbol{v_m}} = \left[\frac{d\overrightarrow{AG}}{dt}\right]_0 = \left[\frac{d\left(\overrightarrow{AB} + \overrightarrow{BG}\right)}{dt}\right]_0 = \overrightarrow{V(G/0)} - \overrightarrow{V(A/0)}$$

h) Draw $\overrightarrow{v_m}$ and give its amplitude. Does it satisfy the condition given in the introduction for the configuration represented in figure 3?

ANNEX 1 – Part 1.	LAST NAME	FIRST NAME

Figure 2. Kinematical model of the 3D mixer



Geometrical data:

$\overrightarrow{AB} = r_1\overrightarrow{z_0}$

$\overrightarrow{BC} = R_2\overrightarrow{z_0}$

$\overrightarrow{CD} = b\,\overrightarrow{z_2}$

$\overrightarrow{CE} = a\,\overrightarrow{x_0}$

$\overrightarrow{EM} = c\overrightarrow{x_4} - d\overrightarrow{z_4}$

$\overrightarrow{EG_4} = e\overrightarrow{x_4} - f\overrightarrow{z_4}$

Graph of links and change of basis diagrams:

ANNEX 1 – Part 1.	LAST NAME	FIRST NAME

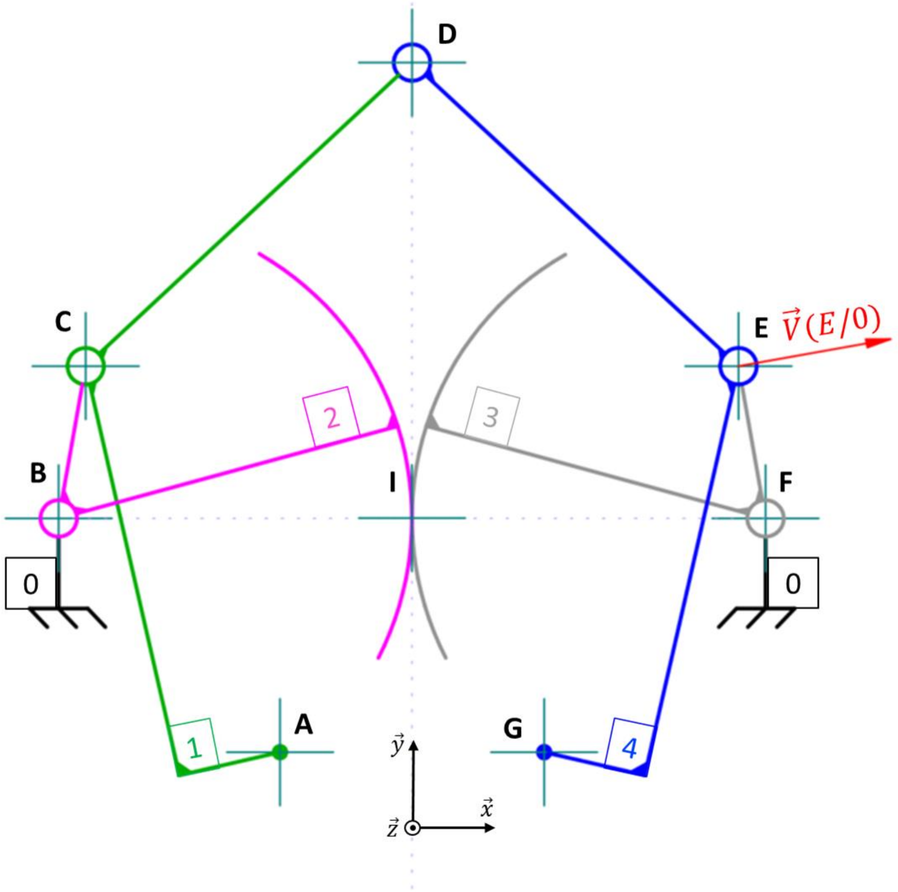


Figure 3 : 2D mixer – Figure for graphical constructions