INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #10 — 10'

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Exercise 1. Let $u : \mathbb{R}^2 \to \mathbb{R}$ and $v : \mathbb{R} \to \mathbb{R}$ be functions of class C^2 and define the function f as

$$f : \begin{array}{ccc} \mathbb{R}^2 & \longrightarrow & \mathbb{R} \\ (x, y) & \longmapsto & u(2x + y, v(x)). \end{array}$$

Compute, for $(x, y) \in \mathbb{R}^2$ the following partial derivatives:

$\partial_1 f(x, y) =$ $2\,\partial_1 u(2x+y, v(x)) + v'(x)\,\partial_2 u(2x+y, v(x))$

$\partial^2_{1,1} f(x, y) =$ $4\,\partial^2_{1,1} u(2x+y, v(x)) + 2\,v'(x)\,\partial^2_{2,1} u(2x+y, v(x) + v''(x)\,\partial_2 u(2x+y, v(x))$
$+ v'(x)\big(2\,\partial^2_{1,2} u(2x+y, v(x)) + v'(x)\,\partial^2_{2,2} u(2x+y, v(x))$

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Exercise 2. Let $n \in \mathbb{N}^*$, $k \in \mathbb{N}^* \cup \{\infty\}$ and let U and V be two open subsets of $\mathbb{R}^n$. Let $\psi : U \to V$. Recall the definition of

"ψ is a C^k-diffeomorphism."

- ψ is a bijection
- ψ is of class C^k on U
- ψ^{-1} is of class C^k on V

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Exercise 3. Let $n \in \mathbb{N}^*$, let U and V be two open subsets of $\mathbb{R}^n$ and let $\varphi : U \to V$ be a C^1-diffeomorphism. Let $y_0 \in V$. Express the Jacobian matrix of φ^{-1} at y_0 in terms of the Jacobian matrix of φ at a well-chosen point.

$J_{y_0}(\varphi^{-1}) =$ $\left(J_{\varphi^{-1}(y_0)}\varphi\right)^{-1}$

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Exercise 4. Recall the Global Inverse Function Theorem.

Let U and V be two open subsets of $\mathbb{R}^n$; let $\varphi : U \to V$
if φ is a bijection such that it is of class C^k on U $(k \geq 1)$, and for all $x \in U$ $D_x\varphi$ is invertible, then φ is a C^k diffeomorphism.

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