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INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #12 — 10'

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Name: CANS Anne-Laure

Exercise 1. Give the radius of convergence of the following power series (no justifications required):

n^{-1} x^n

$$\sum_{n=0}^{+\infty} \frac{2n+1}{n^2+1} x^n$$

$\sim \left| \frac{2n+3}{(n+1)^2+1} \cdot \frac{n^2+1}{2n+1} \cdot \frac{x^{n+1}}{x^n} \right| \to |x|$

$1/n$ n x

$R =$ 1

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$$\sum_{n=0}^{+\infty} \left(\frac{2n+1}{n+1}\right)^n x^n$$

$\frac{2n+1}{n+1} x < 1 \Leftrightarrow (2n+1)x < n+1$

$\Leftrightarrow x < \frac{n+1}{2n+1} \to \frac{1}{2}$

$R = \frac{1}{2}$

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$$\sum_{n=0}^{+\infty} a_n x^n$$

where $(a_n)_{n\in\mathbb{N}}$ is such that $\forall n \in \mathbb{N}$, $3n \leq a_n \leq 4n^2$.

$R =$ 1

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Exercise 2. Let f be a function defined by a power series of radius of convergence $R > 0$, say

$$f(x) = \sum_{n=0}^{+\infty} a_n x^n$$

for some real sequence $(a_n)_{n\in\mathbb{N}}$. Write the following expression as a power series:

$\forall x \in (-R, R),\ (3+2x)f'(x) - f(x) = \sum_{n=0}^{+\infty} \left[3a_{n+1}(n+1) + (2n-1)a_n \right] x_n$

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Exercise 3. Let $(a_n)_{n\in\mathbb{N}}$ be a sequence (of real or complex numbers) such that the radius of convergence of the power series $\sum_{n=0}^{+\infty} a_n x^n$ is R_a. Let $(b_n)_{n\in\mathbb{N}}$ be a sequence such that

$$\forall n \in \mathbb{N},\ |b_n| \leq |a_n|.$$

What can we conclude about the radius of convergence R_b of the power series $\sum_{n=0}^{+\infty} b_n x^n$? No justifications required.

$R_b =$ $R_b > R_a$

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