18

INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #14 — 12'

May 15, 2018

Name: CANS Anne-Laure

Exercise 1. Let $\varphi : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ be the symmetric bilinear form on $\mathbb{R}^3$ such that the matrix of φ in the standard basis std $= (e_1, e_2, e_3)$ of $\mathbb{R}^3$ is:

$$[\varphi]_{\text{std}} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 2 \\ 0 & 2 & 0 \end{pmatrix}.$$

1. Determine the value of $\varphi(e_2, e_2)$.

$\varphi(e_2, e_2) = -1$

2

2. Is φ a dot product on $\mathbb{R}^3$? justify your answer.

φ is a dot product iff it is positive definite yet $\varphi(e_2, e_2) < 0 \Rightarrow$ NO

2

3. Give a basis of the orthogonal of $(1, 1, 1)$ with respect to φ.

$(1, 1, -2)$

$$\varphi(e_2, e_2) = (0\ 1\ 0)\begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 2 \\ 0 & 2 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = (0\ 1\ 0)\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = -1$$

$$\varphi((x,y,z),(1,1,1)) = (x\ y\ z)\begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 2 \\ 0 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = (x\ y\ z)\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

$$= 2x + 2y + 2z.$$

$$\Rightarrow (1,1,1)^{\perp\varphi} = \{(x,y,z) \in \mathbb{R}^3 \mid x + y + z = 0\}$$

2

Exercise 2. Let $\varphi : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ be the symmetric bilinear form on $\mathbb{R}^2$ such that the matrix of φ in the standard basis std $= (e_1, e_2)$ of $\mathbb{R}^2$ is:

$$A = [\varphi]_{\text{std}} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}.$$

Let

$$u_1 = (1, 1), \qquad u_2 = (1, -1).$$

You're given that $\mathscr{B} = (u_1, u_2)$ is a basis of $\mathbb{R}^2$. We denote by $B = [\varphi]_{\mathscr{B}}$ the matrix of φ in the basis $\mathscr{B}$, and by q the quadratic form associated with φ.

1. Explicit the change of basis matrix $P = [\mathscr{B}]_{\text{std}}$.

$P = [\mathscr{B}]_{std} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ 1

2. Give a relation between A, B and P. $[\varphi]_{\mathscr{B}} = {}^t[\mathscr{B}]_{std} \, [\varphi]_{std} \, [\mathscr{B}]_{std}$

$B = {}^tPAP$ 2

3. Explicit the matrix B.

$B = [\varphi]_{\mathscr{B}} = \begin{pmatrix} 6 & 0 \\ 0 & 2 \end{pmatrix}$ 2

4. Is the basis $\mathscr{B}$ an orthogonal basis of $\mathbb{R}^2$ with respect to φ? justify your answer.

$\varphi(u_1, u_2) = 0$ hence yes 2

5. Let $v \in \mathbb{R}^2$ and let $[v]_{\mathscr{B}} = \begin{pmatrix} x \\ y \end{pmatrix}$ be the coordinates of v in $\mathscr{B}$. Give the expression of $q(v)$ in terms of x and y.

$q(v) = 6x^2 + 2y^2$ 2

6. Is φ an inner product on $\mathbb{R}^2$? justify your answer.

$\forall (x,y)^2 \in \mathbb{R},\ q((x,y)) \geq 0$ and $q((x,y)) = 0 \iff (x,y) = (0,0)$ hence yes 3

$$\mathbb{B} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 3 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 0 \\ 0 & 2 \end{pmatrix}$$

$(1,1)$

$$(1, 1)\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} +1 \\ -1 \end{pmatrix} = 0$$

$q(v) =$