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SCAN 2 — Quiz #3 — 10'

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19/20

Exercise 1. Is the following improper integral convergent or divergent?

$$I = \int_0^{+\infty} \frac{t + t^3}{t^5 + \arctan t}\, dt.$$

Justify your answer (as *concisely* as possible).

Improper at 0 and at $+\infty$

<u>At $+\infty$:</u> $\arctan t \underset{t\to+\infty}{\sim} \frac{\pi}{2} \Rightarrow \frac{t+t^3}{t^5+\arctan t} \underset{t\to+\infty}{\sim} \frac{1}{t^2}$, $\alpha = 2 > 1$

So $\int_1^{+\infty} \frac{t+t^3}{t^5+\arctan t}\, dt$ CV at $+\infty$.

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At 0 : $\frac{t+t^3}{t^5+\arctan t} \underset{t\to 0}{\sim} \frac{t}{t} = 1$, since $t^3 = o(t)$ and $\arctan(t) \underset{t\to 0}{\sim} t$ $\Rightarrow t^5 = o(\arctan t)$

So $\int_0^1 \frac{t+t^3}{t^5+\arctan t}\, dt$ CV at 0 $\boxed{\Rightarrow I \text{ converges}}$

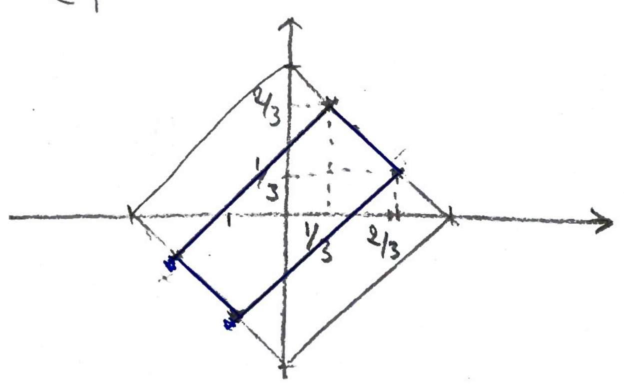
Exercise 2. Let

$$N : \begin{array}{ccc} \mathbb{R}^2 & \longrightarrow & \mathbb{R}_+ \\ (x,y) & \longmapsto & |2x-y| + |x-2y|. \end{array}$$

You're given that N is a norm on $\mathbb{R}^2$. Plot the unit ball of N. No justifications required.

$\varphi : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$
$(x,y) \longmapsto (2x-y, x-2y) \Rightarrow N(x,y) = \|\varphi(x,y)\|_1$

$[\varphi]_{std} = \begin{pmatrix} 2 & -1 \\ 1 & -2 \end{pmatrix} \Rightarrow [\varphi^{-1}]_{std} = -\frac{1}{3} \begin{pmatrix} -2 & 1 \\ -1 & 2 \end{pmatrix}$



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