INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #4 — 10' *October 12, 2017*

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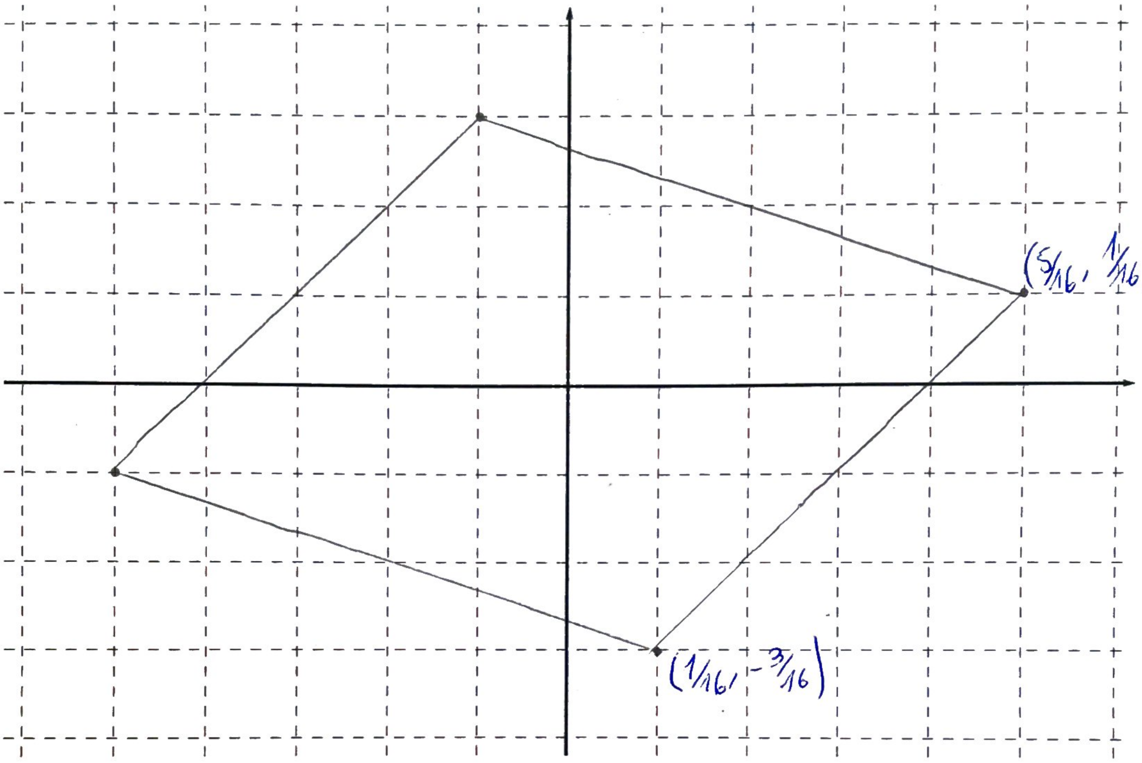
Exercise 1. Let

$$N: \begin{array}{ccc} \mathbb{R}^2 & \longrightarrow & \mathbb{R}_+ \\ (x,y) & \longmapsto & |x-5y|+|3x+y| \end{array}$$

You're given that N is a norm on $\mathbb{R}^2$. Plot the unit ball of N. No justifications required.



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Exercise 2. Let $(E, \|\cdot\|_E)$, $(F, \|\cdot\|_F)$, $(G, \|\cdot\|_G)$ be three normed vector spaces. Let U be an open subset of E and V be an open subset of F. Let $u : U \to V$ and $v : V \to G$ be two functions, and let $x_0 \in U$. The following text is the chain rule theorem. Fill-in the blank:

If u is differentiable at x_0 and v is differentiable at $u(x_0)$ then $v \circ u$ is differentiable at x_0 and

$$D_{x_0}(v \circ u) = D_{u(x_0)} v \circ D_{x_0} u$$

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Exercise 3. Let

$$f: \begin{array}{ccc} \mathbb{R}^2 & \longrightarrow & \mathbb{R} \\ (x,y) & \longmapsto & 3x^2+2y \end{array}$$

and let $(x_0, y_0) \in \mathbb{R}^2$. You're given that f is differentiable at (x_0, y_0). Determine the differential $\mathrm{d}_{(x_0,y_0)} f$ of f at (x_0, y_0). No justifications required.

$(d_{x_0,y_0}) : \varphi : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, $(h,k) \longmapsto d_{x_0,y_0}$

$$d_{(x_0,y_0)} f = 6x_0 h + 2k$$

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