INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #5 — 10'

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Name: MELLOUK Chouaïb

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Exercise 1. Let

$$f: \begin{array}{ccc} \mathbb{R}^2 & \longrightarrow & \mathbb{R}^2 \\ (x, y) & \longmapsto & (3x^2 - 2y^2, 5xy). \end{array}$$

Let $(x_0, y_0) \in \mathbb{R}^2$. You're given that f is differentiable at (x_0, y_0) and you don't have to prove this fact. Determine the differential $D_{(x_0,y_0)}f$ of f at (x_0, y_0). No justifications required.

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$$D_{(x_0,y_0)}f : \begin{array}{ccc} \mathbb{R}^2 & \longrightarrow & \mathbb{R}^2 \\ (h,k) & \longmapsto & (6x_0 h - 4y_0 k, 5x_0 k + 5y_0 h) \end{array}$$

Exercise 2. Let $(E, \|\cdot\|_E)$ and $(F, \|\cdot\|_F)$ be two normed vector spaces, let U be an open subset of E, let $h: U \to F$ be a function, and let $p_0 \in U$.

1. Let $a \in E$. Recall the definition of the directional derivative of h at p_0 in the direction a (assuming that it exists):

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$$\nabla_a h(p_0) = \lim_{k \to 0} \frac{h(p_0 + ka) - h(p_0)}{k}$$

2. We assume that h is differentiable at p_0. We know that all the directional derivatives of h at p_0 exist. Give the relation between the directional derivatives of h at p_0 and the differential of h at p_0. No justifications required.

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$$\forall a \in E,\ \nabla_a h(p_0) = D_{p_0} h(a)$$

Exercise 3. Let

$$f: \begin{array}{ccc} \mathbb{R}^2 & \longrightarrow & \mathbb{R} \\ (x, y) & \longmapsto & xy - 2x - y. \end{array}$$

Determine the directional derivative of f at $(1, 1)$ in the direction $(1, 2)$. No justifications required.

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$$\nabla_{(1,2)}f(1,1) = \lim_{t \to 0} \frac{f(1+t, 1+2t) - f(1,1)}{t} = \lim_{t \to 0} \frac{2t^2 - t}{t} \quad ?$$

$$= \lim_{t \to 0} 2t - 1 = -1$$