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20/20

Exercise 1. Find all functions $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ of class C^1 such that

$$(*) \quad \partial_2 f + 4f = 0.$$

No justifications required.

the general solution of class C^1 of $(*)$ is :
 $f : \mathbb{R}^3 \rightarrow \mathbb{R}$
 $(x, y, z) \mapsto A(x, z) e^{-4y}$
 where $A : \mathbb{R}^2 \rightarrow \mathbb{R}$ is of class C^1

Exercise 2. Let f be the function defined by

$$f : \mathbb{R}^3 \rightarrow \mathbb{R}$$

$$(a, b, c) \mapsto ab^2 e^{bc}.$$

Let $(x, y, z) \in \mathbb{R}^3$. Compute (please mind the name of the variables):

$$\partial_{2,3}^2 f(x, y, z) = 3xy^2 e^{yz} + xy^3 z e^{yz}$$

Exercise 3. Let f be the function defined by

$$f : \mathbb{R}^3 \rightarrow \mathbb{R}$$

$$(x, y, z) \mapsto x^3 - 2xy + yz + z^2.$$

Let \mathcal{S} be the surface in \mathbb{R}^3 of equation

$$\mathcal{S} : f(x, y, z) = -1.$$

1. Give the gradient vector of f at $(1, 1, -1)$.

$$\vec{\nabla} f(1, 1, -1) = \vec{e}_1 - 3\vec{e}_2 - \vec{e}_3$$

2. Deduce an equation of the tangent plane (P) to \mathcal{S} at $(1, 1, -1)$. You don't need to check that $(1, 1, -1) \in \mathcal{S}$. No justifications required.

$$(P) : x - 3y - z = -1$$