INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #8 — 10'

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20

Exercise 1. Let f be the function defined by

$$f : \begin{array}{ccc} \mathbb{R}^3 & \longrightarrow & \mathbb{R} \\ (x, y, z) & \longmapsto & y^3 - 2xy + xz + z^2. \end{array}$$

Let $\mathscr{S}$ be the surface in $\mathbb{R}^3$ of equation

$$\mathscr{S} \ : \ f(x, y, z) = 1.$$

1. Give the gradient vector of f at $(1, 1, -2)$.

$\vec{\nabla} f(1, 1, -2) = -4\vec{e_1} + \vec{e_2} - 3\vec{e_3}$

4

2. Deduce an equation of the tangent plane (P) to $\mathscr{S}$ at $(1, 1, -2)$. You don't need to check that $(1, 1, -2) \in \mathscr{S}$. No justifications required.

(P) : $-4x + y - 3z = 3$

3

Exercise 2. Let f be the function defined by

$$f : \begin{array}{ccc} \mathbb{R}^3 & \longrightarrow & \mathbb{R}^2 \\ (x, y, z) & \longmapsto & (2x^2y + e^z, xe^{yz}). \end{array}$$

1. Determine the Jacobian matrix $J_{(-1,1,0)}f$ of f at $(-1, 1, 0)$.

$$J_{(-1,1,0)}f = \begin{pmatrix} -4 & 2 & 1 \\ 1 & 0 & -1 \end{pmatrix}$$

5

2. Deduce the value of the differential of f at $(-1, 1, 0)$ evaluated at $(-1, 2, -1)$.

$D_{(-1,1,0)}f(-1, 2, -1) = (7, 0)$

3

Exercise 3. We denote by (e_1', e_2', e_3') is the dual basis of $\mathbb{R}^3$. Is the differential form ω defined on $\mathbb{R}^3$ by

$$\forall (x, y, z) \in \mathbb{R}^3, \ \omega_{(x,y,z)} = yz\, e_1' + (xy + z)\, e_2' + xy\, e_3'$$

a closed differential form? Justify your answer (as concisely as possible).

$\frac{\partial (yz)}{\partial y} = z \neq \frac{\partial (xy+z)}{\partial x} = y$

Hence the differential form ω is not a closed differential form.

5