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SCAN 2 — Quiz #9 — 10' — *November 23, 2017*

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Exercise 1. Let $u : \mathbb{R}^2 \to \mathbb{R}$, $v : \mathbb{R}^3 \to \mathbb{R}$, $f : \mathbb{R}^2 \to \mathbb{R}$ be functions of class C^1. We define

$$\begin{array}{rccc} g: & \mathbb{R}^2 & \longrightarrow & \mathbb{R} \\ & (x,y) & \longmapsto & f\big(u(y,x)+y\,,\, v(x,y,xy)\big). \end{array}$$

Let $(x,y) \in \mathbb{R}^2$. Compute the first-order partial derivatives of g at (x,y).

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$\partial_1 g(x,y) = \partial_2 u(y,x) \cdot \partial_1 f\big(u(y,x)+y, v(x,y,xy)\big) + \big[\partial_1 v(x,y,xy) + y\partial_3 v(x,y,xy)\big] \cdot \partial_2 f\big(u(y,x)+y, v(x,y,$

$\partial_2 g(x,y) = \big(\partial_1 u(y,x) + 1\big) \cdot \partial_1 f\big(u(y,x)+y, v(x,y,xy)\big) + \big[\partial_2 v(x,y,xy) + x\partial_3 v(x,y,xy)\big] \cdot \partial_2 f\big(u(y,x)+y, v(x,y,xy)\big)$

Exercise 2. Let U be an open subset of $\mathbb{R}^n$ (with $n \in \mathbb{N}^*$) and let $v : U \to \mathbb{R}$ be a function of class C^2. Let $p_0 \in U$. Recall the second-order Taylor–Young formula for v at p_0.

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$$f(p_0+v) \underset{v \to 0}{=\!=} f(p_0) + J_{p_0} f \cdot [v]_{std} + \frac{1}{2}\,{}^t[v]_{std}\, H_{p_0} f \cdot [v]_{std} + o(\|v\|_\infty^2)$$

Exercise 3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function of class C^2 such that

$$f(1,-2) = 2, \qquad \partial_1 f(1,-2) = -1, \qquad \partial_2 f(1,-2) = 2,$$
$$\partial^2_{1,1} f(1,-2) = 4, \qquad \partial^2_{1,2} f(1,-2) = 3, \qquad \partial^2_{2,2} f(1,-2) = -1.$$

Give the second order Taylor–Young expansion of f at $(1,-2)$.

Let $(h,k) \in \mathbb{R}^2$.

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$$f((1,-2)+(h,k)) \underset{h \to 0}{=\!=} 2 - h + 2k + 2h^2 + 3kh - \frac{1}{2}k^2 + o(\|(h,k)\|^2)$$

$$J_{1,-2} f = \begin{pmatrix} -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} h \\ k \end{pmatrix} = -h + 2k$$

$$H_{1,-2} f = \begin{pmatrix} 4 & 3 \\ 3 & -1 \end{pmatrix} \Longrightarrow \begin{pmatrix} 4 & 3 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} 4h + 3k \\ 3h - k \end{pmatrix} = 4h^2 + 3kh + 3hk - k^2$$

$(h\ k)\cdot$

$$= 2h^2 + 3kh - \frac{1}{2}k^2$$