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INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #10 — 12'

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Exercise 1. Let $(E, \|\|_E)$ and $(F, \|\|_F)$ be two normed vector spaces, let U be an open subset of E and let V be an open subset of F. Let $f : U \to V$ be a bijection such that f is differentiable at every point of U and f^{-1} is differentiable at every point of V. Recall the relation between the differential of f^{-1} at a point $y_0 \in V$ and the differential of f at a well-chosen point. No justifications required.

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$$D_{f(x_0)}(f^{-1}) = (D_{x_0} f)^{-1}$$

Exercise 2. Let

$$\begin{array}{rccc} \varphi : & \mathbb{R}^2 & \longrightarrow & \mathbb{R}^2 \\ & (x, y) & \longmapsto & (x + y^2, x^2 - 3y). \end{array}$$

You're given that φ is well-defined, of class C^∞, and you don't have to justify any of these properties.
On the back of this sheet are two pictures. The first one represents the domain of φ (i.e., $\mathbb{R}^2$) and the second one represents the codomain of φ (also $\mathbb{R}^2$).
On the first picture we have represented the point $A(2, 1)$ and a little house. In fact the house is very very small, so the picture is not exactly to scale.
You're asked to determine the image of A by φ, and plot it on the second picture. You're also asked to plot the image of the little house by φ (still on the second picture). The overall scale of the image is not relevant. What's important, though, is to preserve the relative scale for the horizontal and vertical axes when plotting the image of the little house.
You're also asked to plot, on the first picture, the coordinates induced by φ, that is, the curves of the form $\varphi(x, y) = (u_0, 0)$ for some $u_0 \in \mathbb{R}$ (that we shall call the u-coordinates) and the curves of the form $\varphi(x, y) = (0, v_0)$ for some $v_0 \in \mathbb{R}$ (that we shall call the v-coordinates). Please use different colors for the u-coordinates and the v-coordinates (and specify which color you used). Plot at least five u-coordinates and five v-coordinates (but no more than 42,000,000 for each).
For your convenience, all the necessary information you need is also included on the back of this sheet.

$x + y^2 = u_0$

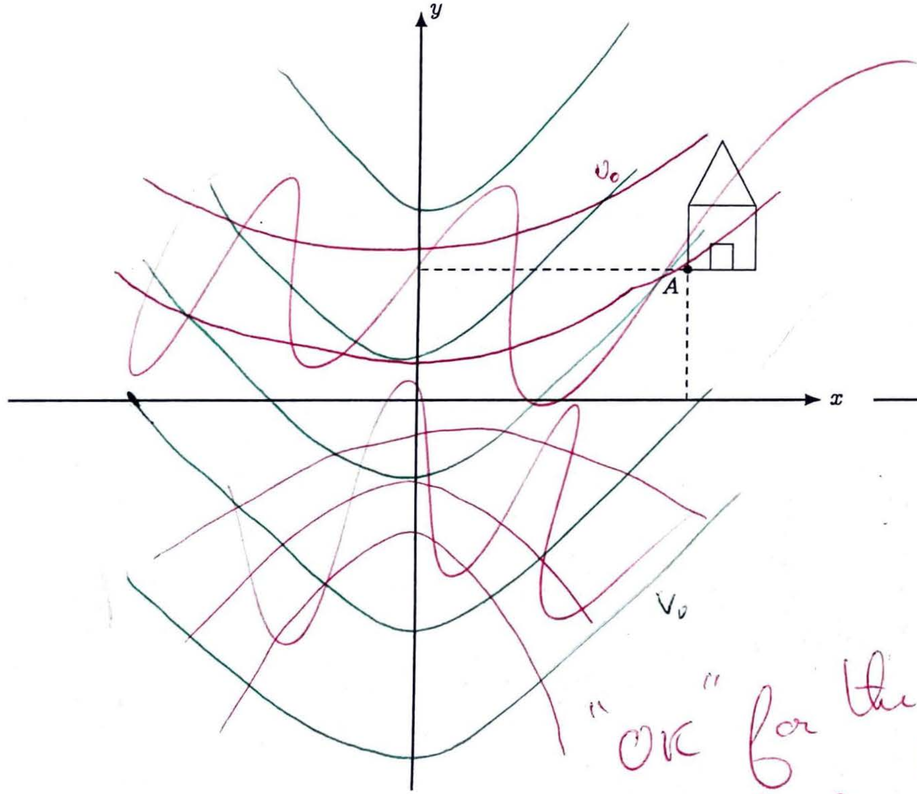
$x^2 - 3y = v_0$

$y = \pm\sqrt{u_0 - x}$

$y = \frac{x^2 - v_0}{3}$

$$\begin{aligned} \varphi : \quad \mathbb{R}^2 &\longrightarrow \mathbb{R}^2 \\ (x, y) &\longmapsto (x + y^2, x^2 - 3y). \end{aligned}$$

$A(2, 1)$



"OK" for the green !

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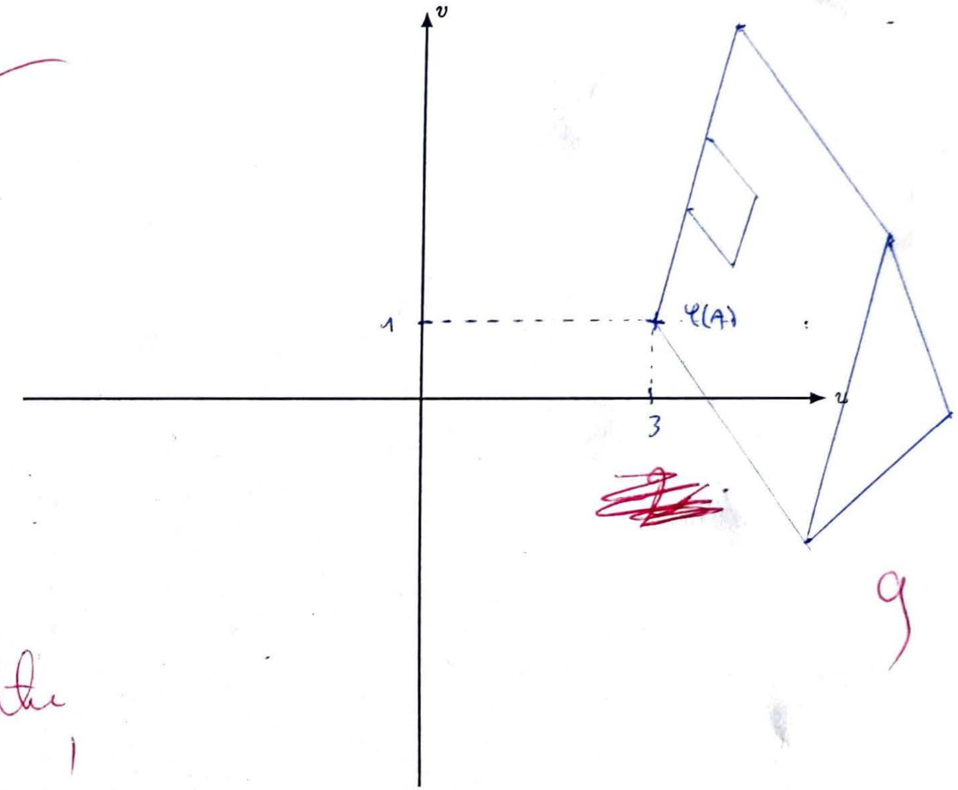

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- Plot $\varphi(A)$ on the second picture;
- Plot, on the second picture, the image of the little house by φ;
- Plot, on the first picture, some u- and v-coordinates (at least 5 for each). Use two different colors.