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INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #11 — 10' *December 13, 2018*

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Exercise 1. State the "baby version" of the Implicit Function Theorem. If you don't really know what we mean by the "baby version," you can state the full theorem. You don't have to state the "moreover" part (about the formula for the derivative or the partial derivatives).

6

Let $f : \mathbb{R}^2 \to \mathbb{R}$ of class C^1
Let $(x_0, y_0) \in \mathbb{R}^2$ st $f(x_0,y_0) = 0$
and $\partial_2 f(x_0,y_0) \neq 0$
Then, there exists an open interval V containg x_0
an open interval W containg y_0
a function $\varphi : V \to W$ of class C^1
such that
$\forall (x,y) \in V \times W$, $(f(x,y) = 0 \iff y = \varphi(x))$

Exercise 2. Let

$$f : \begin{array}{ccc} \mathbb{R}^2 & \longrightarrow & \mathbb{R} \\ (x,y) & \longmapsto & x^2y + xy^2 - 2, \end{array}$$

and $(x_0, y_0) = (1,1)$.

1. Check that you can apply the (baby version of the) Implicit Function Theorem to f at (x_0, y_0).

4

· f is of class C^1
· $f(1,1) = 1 + 1 - 2 = 0$ and $\partial_2 f(x,y) = x^2 + 2yx \cancel{-2}$
· hence $\underset{x_0, y_0}{(1,1)} \in \mathbb{R}^2$ st $f(1,1) = 0$ and $\partial_2 f(1,1) \neq 0$
hence $\partial_2 f(1,1) = 1 + 2 \cancel{-2} = \cancel{1}\ 3 \neq 0$

2. What conclusion do you obtain? (do not state the "moreover part" of the conclusion). Please denote by φ the function given by the conclusion of the theorem. You shouldn't have any "f" in your answer; use the explicit form of f if you need it.

4

hence, we can conclude that:
There exists an open interval V containing $x_0 = 1$
an open interval W containing $y_0 = 1$
a function $\varphi : V \to W$ of class C^1
such that $\forall (x,y) \in V \times W$, $(x^2y + xy^2 - 2 = 0 \iff y = \varphi(x))$

3. Now state the moreover part, about the derivative of φ (you shouldn't have any f in your answer, use the explicit form of f if you need it).

2

Moreover:
$\forall x \in V$ $\varphi'(x) = \dfrac{-\partial_1 f(x,\cancel{y})}{\partial_2 f(x,\cancel{y})} = \dfrac{-2x\cancel{y} + \cancel{y^2} - \cancel{x}}{x^2 + 2x\cancel{y} - \cancel{2}}$

NOT y but $\varphi(x)$ instead!