$\frac{x}{2} < 1$ $\left(\frac{x}{2}\right)^n \cdot \frac{1}{n}$ $\frac{x^{n+1} 2^n \cdot n}{2^{n+1}(n+1) x^n} = \frac{x\, n}{2(n+1)}$ $\left(\frac{x}{2}\right)^n \frac{1}{n} < \left(\frac{x}{2}\right)^n$

$x < 2$

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$\frac{x}{2} < 1$ $\left(\frac{x}{2}\right)^n$

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SCAN 2 — Quiz #12 — 10' January 23, 2019

$x < 2$.

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Exercise 1. Is the following series convergent or divergent? (justify as concisely as possible)

$$\sum_n \frac{1+\sin(n)}{n^2}.$$

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$\sum_n \frac{1+\sin(n)}{n^2} \leq 2 \sum_n \frac{1}{n^2}$

we know by Riemann series that $\frac{1}{n^2}$ is a GTOACS because $\overset{\alpha}{2} > 1$

hence $2 \sum_n \frac{1}{n^2}$ CV. Hence by the comparison test $\sum_n \frac{1+\sin(n)}{n^2}$ CV

Exercise 2. Let $x \in \mathbb{R}_+^*$. Fill in the blank (no justifications required):

4

The series $\sum_n \frac{x^n}{2^n n}$ converges $\iff$ $x < 2$

Exercise 3. Let $\alpha \in \mathbb{R}$ and $q \in \mathbb{C}$. Fill in the blanks (no justifications required):

3

The series $\sum_n \frac{1}{n^\alpha}$ converges $\iff$ $\alpha > 1$

3

The series $\sum_n q^n$ converges $\iff$ $|q| < 1$

Exercise 4. Is the following series convergent or divergent? (justify as concisely as possible)

$\ln(1+X) \underset{X \to 0}{\sim} X$

$$\sum_n \ln\left(1+\frac{1}{3^n}\right).$$

4

set $X = \frac{1}{3^n}$? hence $\sum_n \ln(1+\frac{1}{3^n}) = \sum_n \ln(1+X)$?

when $n \to +\infty$, $\frac{1}{3^n} \to 0$ hence when $n \to +\infty$ $X \to 0$

$\ln(1+X) \underset{X \to 0}{\sim} X = \frac{1}{3^n}$ and $\left(\frac{1}{3}\right)^n$ is the GTOA geometric CS

Hence by the equivalent test $\sum_n \ln(1+\frac{1}{3^n})$ CV