$\ln(1+x) \to 0$ $\sum_{n=0}^{+\infty} \frac{f^{(k)}(0)}{k!} x^k$

$\hookrightarrow \frac{1}{1+x} \to 1$ $0 + x - \frac{x^2}{2}$ 18/20

$-\frac{1}{(1+x)^2} \to -1$

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SCAN 2 — Quiz #13 — 12' February 26, 2019

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Exercise 1. Determine the radius of convergence of the following power series. No justifications required:

- $\sum_{n=0}^{+\infty} \frac{z^n}{n+1}$.

$\boxed{R = 1 \quad 2}$

- $\sum_{n=0}^{+\infty} \frac{z^n}{n!}$.

$\boxed{R = +\infty \quad 2}$

- $\sum_{n=0}^{+\infty} n! z^n$.

$\boxed{R = 0 \quad 2}$

- $\sum_{n=0}^{+\infty} \frac{z^{2n}}{3^n}$. $= \left(\frac{z^2}{3}\right)^n$ $\quad \frac{z^2}{3} < 1 \iff z^2 < 3$ $\quad |z| < \sqrt{3}$

$\boxed{R = \sqrt{3} \quad 3}$

Exercise 2. Determine the values of $\alpha \in \mathbb{R}_+^*$ such that the following series converges:

$$(S) \qquad \sum_n \ln\left(1 + \frac{(-1)^n}{n^\alpha}\right) - \underbrace{\frac{(-1)^n}{n^\alpha}}_{}$$ cv for $\alpha > 0$ by alt ser. test

No justifications required.

$\boxed{(S) \text{ converges} \iff \alpha > \frac{1}{2} \quad 6}$

Exercise 3. Let $\sum_{n=0}^{+\infty} a^n z^n$ be a power series that converges at $z = 1 + 2i$. What can you conclude about R? No justifications required.

$|z| = \sqrt{3}$ X $\sqrt{5}$

$\boxed{R \geq \cancel{\sqrt{3}} \ \sqrt{5} \quad 3/5}$

$$\ln\left(1 + \frac{(-1)^n}{n^\alpha}\right) \underset{n \to +\infty}{=} 0 + \underbrace{\frac{(-1)^n}{n^\alpha}}_{\text{cv for } \alpha > 0 \text{ alt. ser. test}} - \underbrace{\frac{1}{2} \frac{1}{n^{2\alpha}}}_{\text{cv for } \alpha > \frac{1}{2}}$$