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INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #14 — 10'

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$e^x = \frac{x^n}{n}$

Exercise 1. Give (without any justifications) the power series expansion of the following expressions, together with the open interval of convergence.

$\forall x \in \mathbb{R}$, $e^{2x} = \sum_{n=0}^{+\infty} \frac{(2x)^n}{n}$ ✓ 3

$\forall x \in \mathbb{R}$, $\sin(x) = \sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ ✓ 3

$\forall x \in \cancel{(-1,1)}$, $\ln(1+3x) = \sum_{n=1}^{+\infty} (-1)^{n-1} \frac{(3x)^n}{n}$ 2

$f'(x) = \sum_{1} n a_n x^{n-1}$ ✓

$= \sum_{0} (n+1) a_{n+1} x^n$ ✓

$\boxed{x f'(x) = \sum_{1} n a_n x^n = \sum_{0} (n+1) a_{n+1} x^{n+1}}$

Exercise 2. Let $R > 0$ and let $f : (-R, R) \to \mathbb{R}$ be a function defined by a power series, say

$$\forall x \in (-R,R),\ f(x) = \sum_{n=0}^{+\infty} a_n x^n.$$

Fill in the blank with the power series expansion (no justifications required):

$(1-x)f'(x) + (2+x)f(x) = \cancel{(1-x)\left(\sum_{n=0}^{+\infty} n a_n x^{n-1}\right) + (2+x)\sum_{n=2}^{+\infty}(a_n x^n)}$

$\cancel{= \sum_{n=0}^{+\infty} \Big(a_n x^n}$

3

$f'(x) = \left(\sum_{n=0}^{+\infty} a_n x^n\right)' = \sum_{n=1}^{+\infty} n a_n x^{n-1} = \sum_{n=0}^{+\infty} (n+1) a_{n+1} x^n$

$x f(x) = \sum a_n x^{n+1}$

$\cancel{= \sum_{n=0}^{+\infty} n a_n x^{n-1}}$

$\cancel{x f'(x) = \sum_{n=0}^{+\infty} n a_n x^n = \sum_{n=0}^{+\infty} (n+1) a_{n+1} x^{n+1}}$

$(1-x)f'(x) + (2+x)f(x) = (1-x)\sum_{n=0}^{+\infty} (n+1) a_{n+1} x^n + (2+x)\sum_{0}^{+\infty} a_n x^n$

$= \sum \Big((n+1) a_{n+1} x^n - (n+1) a_{n+1} x^{n+1} + 2a_n x^n + a_n x^{n+1} \Big)$

$= \sum_{n=0}^{+\infty} \Big((n+1) a_{n+1} + 2a_n \Big) x^n + \Big(-(n+1) a_{n+1} + a_n \Big) x^{n+1}$

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