20/20

INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #15 — 11' — March 26, 2019

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Exercise 1. Determine the polar form φ of the quadratic form q on $\mathbb{R}^2$ defined by

$$\begin{array}{rccc} q: & \mathbb{R}^2 & \longrightarrow & \mathbb{R} \\ & (x,y) & \longmapsto & 2x^2 - 3xy + 6y^2. \end{array}$$

3

Let, $\varphi: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$

$\varphi((x_1,y_1),(x_2,y_2)) = 2x_1x_2 - \frac{3}{2}(x_1y_2 + x_2y_1) + 6y_1y_2$

Exercise 2. Let E be a vector space over $\mathbb{R}$ and let $q: E \to \mathbb{R}$ be a quadratic form. Express the polar form φ of q in terms of q only (give only one of the three polarization identities).

3

$\forall u, v \in E,\ \varphi(u,v) = \frac{1}{2}\left(q(u+v) - q(u) - q(v)\right)$

Exercise 3. Let φ be the symmetric bilinear form on $\mathbb{R}^3$ defined by

$$\begin{array}{rccc} \varphi: & \mathbb{R}^3 \times \mathbb{R}^3 & \longrightarrow & \mathbb{R} \\ & ((x_1,y_1,z_1),(x_2,y_2,z_2)) & \longmapsto & x_1x_2 + 2y_1z_2 + 2y_2z_1. \end{array}$$

1. Are the following asserions true or false?

$2 + 2\times1\times(-2) + 2\times1\times1 = 2 - 4 + 2 = 0$

3 • $(2,1,1) \perp_\varphi (1,1,-2)$ TRUE

3 • $\forall y, z \in \mathbb{R},\ (1,0,1) \perp_\varphi (0,y,z)$ False

$0 + 2\times0 + y \neq 0$

2. Fill in the blank:

$a + 2(a+2) + 2(a+1)$

4

$\forall a \in \mathbb{R},\ \left((1,1,1) \perp_\varphi (a, a+1, a+2) \iff a = -\frac{6}{5}\right)$

$a + 2a + 4 + 2a + 2 = 0$

$5a + 6 = 0$

$a = -\frac{6}{5}$

3. Give an expression of the quadratic form q associated with φ:

2

$\forall (x,y,z) \in \mathbb{R}^3,\ q(x,y,z) = x^2 + 4yz$

4. Determine the matrix A such that

$$\forall (x_1,y_1,z_1),(x_2,y_2,z_2) \in \mathbb{R}^3,\ \varphi((x_1,y_1,z_1),(x_2,y_2,z_2)) = \begin{pmatrix} x_1 & y_1 & z_1 \end{pmatrix} A \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}.$$

2

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & a \end{pmatrix}$$