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INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #16 — 12' *April 30, 2019*

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Exercise 1. Let φ be the symmetric bilinear form on $\mathbb{R}^2$ the matrix of which in the standard basis std $= (e_1, e_2)$ of $\mathbb{R}^2$ is

$$[\varphi]_{\mathrm{std}} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Let

$$u_1 = (1,1), \qquad \text{and} \qquad u_2 = (1,-1),$$

and let $\mathscr{C} = (u_1, u_2)$. You're given that $\mathscr{C}$ is a basis of $\mathbb{R}^2$.
Use the change of basis formula to determine the matrix $[\varphi]_{\mathscr{C}}$ of φ in $\mathscr{C}$.

$[\varphi]_{\mathscr{C}} = \begin{bmatrix} 2 & 2 \\ 2 & -2 \end{bmatrix}$. 5

Exercise 2. Let φ be the symmetric bilinear form on $\mathbb{R}^3$ such that its matrix in the standard basis std of $\mathbb{R}^3$ is

$$[\varphi]_{\mathrm{std}} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

You're given that φ is an inner product on $\mathbb{R}^3$. Let

$$e_1 = (1,1,0) \qquad \text{and} \qquad e_2 = (0,0,1),$$

and let

$$F = \mathrm{Span}\{e_1, e_2\}.$$

Give an orthogonal basis (with respect to φ) $\mathscr{B}$ of F.

$\mathscr{B} = \{(1,1,0), (-1,-1,0)\}$ ← the 2 vectors aren't independent! X

and deduce an orthonormal basis (with respect to φ) $\mathscr{B}'$ of F:

$\mathscr{B}' = \{(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0), (-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0)\}$ ⚠ with respect to φ

Exercise 3. Let (E, φ) be a pre-Hilbert space with norm $\|\cdot\|$. Recall the Pythagorean Theorem.

let $u, v \in E$ $\quad u \perp_\varphi v \iff \|u+v\|^2 = \|u\|^2 + \|v\|^2$ 5