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INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #17 — 12'

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Exercise 1. Let $A = \begin{pmatrix} 0 & 2 & -2 \\ 2 & -3 & 4 \\ -2 & 4 & -3 \end{pmatrix}$.

1. Briefly explain why there exists a real diagonal matrix D and an orthogonal matrix P such that $A = PD\,{}^tP$.

~~[illegible]~~ Because it is a symmetric matrix with real coefficients A? — 3

2. Find a real diagonal matrix D and an orthogonal matrix P such that $A = PD\,{}^tP$.

$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -8 \end{pmatrix}$ ✓ — 6

$P = \cancel{\begin{pmatrix} 0 & -4 & -1 \\ 1 & -1 & 2 \\ 1 & +1 & -2 \end{pmatrix}} = \begin{pmatrix} 0 & -\frac{4}{3\sqrt{2}} & -1/3 \\ 1/\sqrt{2} & -\frac{1}{3\sqrt{2}} & 2/3 \\ \frac{1}{\sqrt{2}} & +\frac{1}{3\sqrt{2}} & -2/3 \end{pmatrix}$ ✓ ✓ — 7

3. What is the signature of A?

sign$(A) = (2, 1)$ — 2

Exercise 2. Let $P \in M_n(\mathbb{R})$. Recall the definition of "P is an orthogonal matrix."

P is an orthogonal matrix if ${}^tPP = I_n$ — 2

(1) → eigen value
$m_1 + = \dim E_1 = \dim - rk$
$= 3 - 1$
$= 2$

$tR = -6 = 1 + 1 + x \quad -8$

$-1 \quad 2 \quad -2$
$2 \quad -4 \quad 4$
$-2 \quad 4 \quad -4$

$P = \begin{pmatrix} 0 & 2 & -2 \\ 0 & -3 & \end{pmatrix}$

$-1x \quad 2y \quad -2z = 0$

$0, 1, 1$

$+4 - 2 - 2 = 0$

$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \\ +1 \end{pmatrix}$

$\sqrt{0, +1^2 + 1^2} = \sqrt{2}$

$\sqrt{1 + 4 + 4}$

9

$16 + 1 + 1$

$= 18$

3×6

9×2