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INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #18 — 12' *June 4, 2019*

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Exercise 1. Let f be the function defined by:

$$f : \begin{array}{ccc} \mathbb{R}^2 & \longrightarrow & \mathbb{R} \\ (x,y) & \longmapsto & x^2y + y^3 + x^2 + 9y^2. \end{array}$$

1. Determine the critical points of f (hint, there are 4 critical points). No justifications required.

$(0,0)$ ✓ $(0,-6)$ ✓ $(\sqrt{15},-1)$ ✓ $(-\sqrt{15},-1)$ ✓

2. For each critical point, determine the Hessian matrix as well as the nature of the critical point (whether it corresponds to a local min, local max, saddle point):

Hessian: $\begin{pmatrix} 2y+2 & 2x \\ 2x & 6y+18 \end{pmatrix}$ ×

q_1: $\begin{pmatrix} 2 & 0 \\ 0 & 18 \end{pmatrix}$ ✓ : sign $(2,0)$. local min

q_2 $\begin{pmatrix} -10 & 0 \\ 0 & -54 \end{pmatrix}$? sign $(0,2)$ local max

q_3 $\begin{pmatrix} 0 & 2\sqrt{15} \\ 2\sqrt{11} & -24 \end{pmatrix}$ sign $(1,1)$ saddle point

q_4 $\begin{pmatrix} 0 & -2\sqrt{15} \\ -2\sqrt{15} & 12 \end{pmatrix}$. saddle point

consistent.

$\begin{cases} 2xy + 2x = 0 \\ x^2 + 3y^2 + 18y = 0 \end{cases}$ $\begin{cases} (2x+1)y = 0 \\ x^2 + 3y(y+6) = 0 \end{cases}$

$\begin{cases} y = 0 \\ x = 0 \end{cases}$ OR $\begin{cases} x = -\frac{1}{2} \\ 3y^2 + 18y = +\frac{1}{4} \end{cases}$

$y^2 + 6y = \frac{1}{12}$

$y(y+6) = \frac{1}{12}$

$y =$

$y^2 = \frac{1}{12} - 6y$

$y = \pm\sqrt{\frac{1}{12} - 6y}$