INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #1 — 11' 19 September 27, 2018

Name: JOSSE Théliau

Exercise 1. You're given that the following improper integral is convergent:

$$I = \int_0^{+\infty} t^2 e^{-t^3}\, dt.$$

improper at $+\infty$

$\int_0^x t^2 e^{-t^3} dt = [$

Determine the value of I (no justifications required).

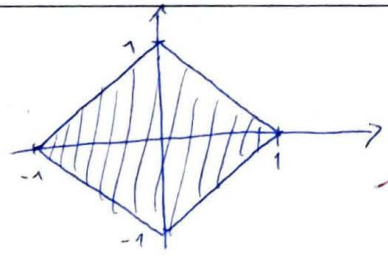
$I =$ ~~2~~ (see behind please)

Exercise 2. Let $E = \mathbb{R}^2$.

1. Recall the definition of the norm $\|\cdot\|_1$ on E.

~~$\|\cdot\|_1$ is the set of vectors s.t $\forall u \in E$, $\|u\| = 1$~~

2. Sketch the closed unit ball $\overline{B}$ of $(E, \|\cdot\|_1)$.



4

Exercise 3. Let E be a vector space over $\mathbb{R}$. Recall the definition of "N is a norm on E."

$N : E \to \mathbb{R}^+$ is a norm on E if it satisfies:

- $\forall u \in E,\ N(u) = 0 \Rightarrow u = 0_E$
- $\forall u \in E,\ \forall \lambda \in \mathbb{K},\ N(\lambda u) = |\lambda| N(u)$
- $\forall u, v \in E,\ N(u+v) \leq N(u) + N(v)$

4

Exercise 4. Let $\alpha \in \mathbb{R}$. Fill in the blanks:

- the improper integral $\int_1^{+\infty} \frac{dt}{t^\alpha}$ converges $\iff$ $\alpha > 1$
- the improper integral $\int_0^1 \frac{dt}{t^\alpha}$ diverges $\iff$ $\alpha \geq 1$
- the improper integral $\int_0^{+\infty} e^{-\alpha t}\, dt$ converges $\iff$ $\alpha > 0$

6