18.5/20

INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #4 — 10' | October 18, 2018

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Exercise 1. Let E be a normed vector space and let $\|\cdot\|$ and $\|\cdot\|'$ be two norms on E. Recall the definition of

"The norms $\|\cdot\|$ and $\|\cdot\|'$ are equivalent."

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$\exists\ \alpha>0,\ \beta>0$ st:

$$\alpha\|\cdot\| \leq \|\cdot\|' \leq \beta\|\cdot\|$$

Exercise 2. Let

$$N:\ \begin{array}{rcl}\mathbb{R}^2 & \longrightarrow & \mathbb{R}_+\\ (x,y) & \longmapsto & |x+2y|+|x+3y|\end{array}$$

You're given that N is a norm on $\mathbb{R}^2$. Plot the closed unit ball of N. No justifications required.

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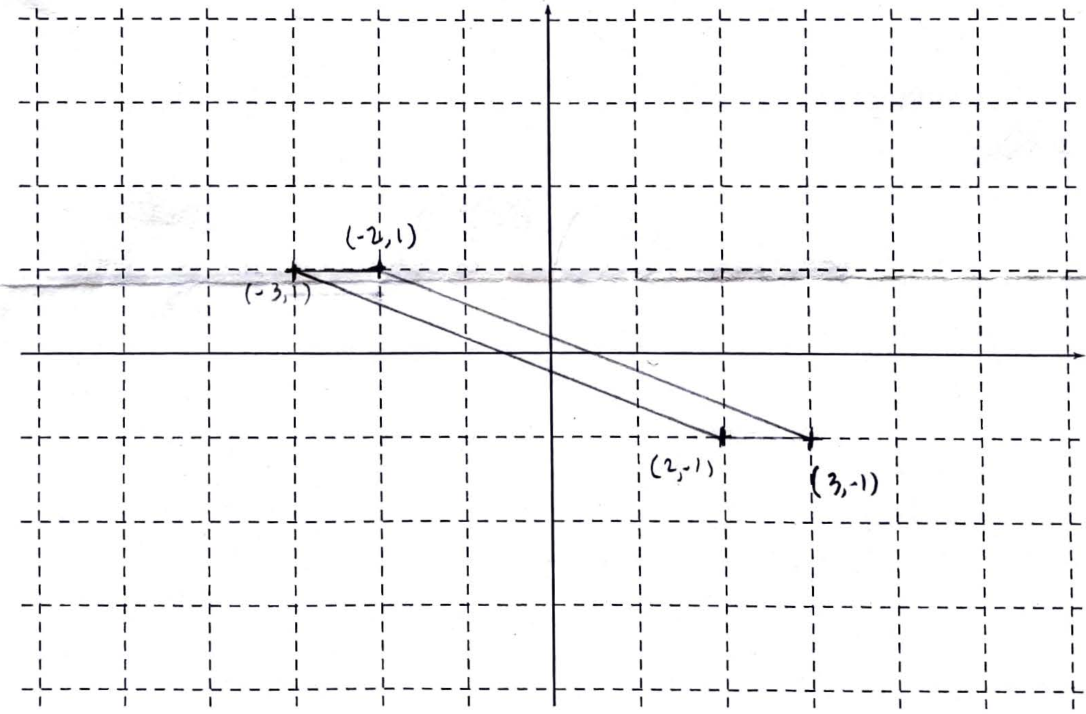

Exercise 3. Let E be a vector space and let $\|\cdot\|$ and $\|\cdot\|'$ be two norms on E such that

$$\forall u\in E,\ \|u\| \leq 2\|u\|'.$$

For $r>0$ we denote by $\overline{B_r}$ the closed ball of $(E,\|\cdot\|)$ centered at 0_E of radius r, and by $\overline{B_r'}$ the closed ball of $(E,\|\cdot\|')$ centered at 0_E of radius r.
Are the following inclusions true? Answer only by "true" or "false." No justifications required.

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- $\overline{B_1}\subset\overline{B_2'}$ False
- $\overline{B_1}\subset\overline{B_{1/2}'}$ False
- $\overline{B_2'}\subset\overline{B_1}$ ~~True~~
- $\overline{B_{1/2}'}\subset\overline{B_1}$ True