INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #6 — 12'

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$1 \times e'_1 + 2 \times e'_2$

Exercise 1. Let

$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}, \quad (x,y) \longmapsto xy - 2x - y.$$

$x - 2$, $y - 1$, $(y-2) + 2 \times (x-1)$, (-1)

Determine the directional derivative of f at $(1,1)$ in the direction $(1,2)$. No justifications required.

$\nabla_{(1,2)} f(1,1) = -1$ ✓

Exercise 2. Let $(E, \|\cdot\|_E)$ and $(F, \|\cdot\|_F)$ be two normed vector spaces, let U be an open subset of E, let $g : U \to F$ be a function, and let $q_0 \in U$.

1. Let $w \in E$. Recall the definition of the directional derivative of g at q_0 in the direction w (assuming that it exists):

$\nabla_w g(q_0) = \lim_{t \to 0} \dfrac{g(q_0 - tw) - g(q_0)}{t}$ ✓

2. We now assume that g is differentiable at q_0. We know that all the directional derivatives of g at q_0 exist. Give the relation between the directional derivatives of g at q_0 and the differential of g at q_0. No justifications required.

$\forall w \in E,\ \nabla_w g(q_0) = d_{q_0} g(w)$ ✓

Exercise 3. Let $n \in \mathbb{N}^*$, let $\mathscr{B} = (e_1, \dots, e_n)$ be the standard basis of $\mathbb{R}^n$ and let $\mathscr{B}' = (e'_1, \dots, e'_n)$ be the dual basis. Let U be an open subset of $\mathbb{R}^n$, let $f : U \to \mathbb{R}$ be a function, let $a \in U$. We assume that f is differentiable at a. Express the differential of f at a in the dual basis $\mathscr{B}'$, in terms of the partial derivatives of f at a. No justifications required.

$d_a f = \partial_1 f(a)\, e'_1 + \cdots + \partial_n f(a)\, e'_n$ ✓

Exercise 4. Let f be the functions defined by

$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}, \quad (y,x) \longmapsto ye^x + x^2.$$

$\cancel{ye^x + 2x}$, e^x | y': e^x, x': $ye^x + 2x$

There are no typos in the definition of f! Let $(u,v) \in \mathbb{R}^2$. Determine the first order partial derivatives of f at (u,v) (you're given that they exist). No justifications required.

$\partial_1 f(u,v) = e^v$

$\partial_2 f(u,v) = ve^v + 2v$ ✓