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INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #7 — 10'

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Exercise 1. Let f be the function defined by

$$f : \begin{array}{ccc} \mathbb{R}^3 & \longrightarrow & \mathbb{R} \\ (x,y,z) & \longmapsto & x^3 - 2xy + yz + z^2. \end{array}$$

Let $\mathscr{S}$ be the surface in $\mathbb{R}^3$ of equation

$$\mathscr{S} : f(x,y,z) = 1.$$

$3x^2 - 2y$: 1
$-2x + z$: -4
$y + 2z$: -3

1. Give the gradient vector of f at $(1,1,-2)$.

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$\vec{\nabla} f(1,1,-2) =$ $\vec{e_1} - 4\vec{e_2} - 3\vec{e_3}$

2. Deduce an equation of the tangent plane (P) to $\mathscr{S}$ at $(1,1,-2)$. You don't need to check that $(1,1,-2) \in \mathscr{S}$. No justifications required.

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(P) : $x - 4y - 3z = \cancel{1}$ 3

Exercise 2. Find all functions $f : \mathbb{R}^3 \to \mathbb{R}$ such that $\partial_1 f$ exists and such that

$$\partial_1 f = 0.$$

No justifications required.

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There exists $g : \mathbb{R}^2 \to \mathbb{R}$ st
$\forall (x,y,z) \in \mathbb{R}^3$, $f(x,y,z) = g(y,z)$

Exercise 3. Let f be the function defined by

$$f : \begin{array}{ccc} \mathbb{R}^3 & \longrightarrow & \mathbb{R} \\ (a,b,c) & \longmapsto & ab\cos(ac). \end{array}$$

c' : $-a^2 b \sin(ac)$
a' : $-2abc\sin(ac) + a^2bc\cos(ac)$

Let $(x,y,z) \in \mathbb{R}^3$. Compute (please mind the name of the variables):

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$\partial^2_{1,3} f(x,y,z) =$ $\partial_1(\partial_3 f(x,y,z)) = -2xy\sin(xz) + x^2yz\cos(xz)$

Exercise 4. Let f be the function defined by

$$f : \begin{array}{ccc} \mathbb{R}^2 & \longrightarrow & \mathbb{R} \\ (x,y) & \longmapsto & \begin{cases} \dfrac{x^3 - y^3}{x^2 + 2y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases} \end{array}$$

$\lim_{h \to 0} \dfrac{f(h,0) - f(0,0)}{h}$

$\dfrac{h^3/h^2}{h}$

You're given that the first-order partial derivatives of f at $(0,0)$ exist, but you're asked to determine them. No justifications required.

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$\partial_1 f(0,0) =$ 1

$\partial_2 f(0,0) =$ $-\frac{1}{2}$