$1, -1, 0$ $\partial_1 f_1(\dots) = 2xy - z\sin(x)$, $\partial_2 f_1(\dots) = x^2$, $\partial_3 f_1(\dots) = \cos x$

-2 1 $\cos 1$

$\partial_1 f_2(\dots) = z\sin(x) + xz\cos x$, $\partial_2 f_2(\dots) = 0$, $\partial_3 f_2(\dots) = x\sin(x)$

0 0 $\sin(1)$

INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #8 — 10' 20 November 22, 2018

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Exercise 1. Let $f : \mathbb{R}^3 \to \mathbb{R}$ and $g : \mathbb{R}^2 \to \mathbb{R}$ be two differentiable functions and let

$$\begin{array}{rccl} h: & \mathbb{R}^2 & \longrightarrow & \mathbb{R} \\ & (x,y) & \longmapsto & g\big(yf(x,x,y), xf(x^2,xy,y^2)\big). \end{array}$$

For $(x,y) \in \mathbb{R}^2$, compute the first order partial derivatives of h at (x,y):

8

$\partial_1 h(x,y) = y(\partial_1 f(x,x,y) + \partial_2 f(x,x,y))\partial_1 g(\dots)$
$+ (f(x^2,xy,y^2) + x(2x\partial_1 f(x^2,xy,y^2) + y\partial_2 f(x^2,xy,y^2)))\partial_2 g(\dots)$

$\partial_2 h(x,y) = (f(x,x,y) + y\partial_3 f(x,x,y))\partial_1 g(\dots)$
$+ (x^2\partial_2 f(x^2,xy,y^2) + 2xy\,\partial_3 f(x^2,xy,y^2))\partial_2 g(\dots)$

Exercise 2. Let f be the function defined by

$$\begin{array}{rccl} f: & \mathbb{R}^3 & \longrightarrow & \mathbb{R}^2 \\ & (x,y,z) & \longmapsto & \big(x^2y + z\cos(x), xz\sin(x)\big). \end{array}$$

1. Determine the Jacobian matrix $J_{(1,-1,0)}f$ of f at $(1,-1,0)$.

$$J_{(1,-1,0)}f = \begin{pmatrix} -2 & 1 & \cos(1) \\ 0 & 0 & \sin(1) \end{pmatrix}$$

2. Deduce the value of the differential of f at $(1,-1,0)$ evaluated at $(-1,2,-1)$.

$$D_{(1,-1,0)}f(-1,2,-1) = J_{(1,-1,0)}f \times \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = \big(4 - \cos(1), -\sin(1)\big)$$

Exercise 3. Let f be the function defined by

$$\begin{array}{rccl} f: & \mathbb{R}^2 & \longrightarrow & \mathbb{R} \\ & (x,y) & \longmapsto & x^3 - xy^2 + y. \end{array}$$

Determine, for $(x,y) \in \mathbb{R}^2$, the Hessian matrix $H_{(x,y)}f$ of f at (x,y):

$$H_{(x,y)}f = \begin{pmatrix} 6x & -2y \\ -2y & -2x \end{pmatrix}$$

$\partial_{1,1}^2 f = \partial_1(\partial_1 f(\dots)) = \partial_1(3x^2 - y^2) = 6x$

$\partial_{2,1}^2 f = \partial_2(\partial_1) = \partial_2(3x^2 - y^2) = -2y$

$\partial_{1,2}^2 f = \partial_1(\cancel{2yx} + y$

$\partial_{2,2} f^2 = -2xy + 1$

$-2x$