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INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #9 — 12'

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Exercise 1. Let U be an open subset of $\mathbb{R}^n$ (with $n \in \mathbb{N}^*$) and let $v : U \to \mathbb{R}$ be a function of class C^2. Let $p_0 \in U$. Recall the second-order Taylor–Young formula for v at p_0 (the general formula, with matrices).

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Let $h \in U$

$$v(p_0+h) \underset{\|h\|\to 0}{=} v(p_0) + J_{(p_0)}v \cdot [h]_{std} + \frac{1}{2} {}^t[h]_{std} H_{(p_0)}v\, [h]_{std} + o(\|h\|^2)$$

Exercise 2. Let $n \in \mathbb{N}^*$ and $k \in \mathbb{N}^* \cup \{+\infty\}$, let V_1 and V_2 be two open subsets of $\mathbb{R}^n$ and let $u : V_1 \to V_2$ be a function. Recall the definition of "u is a C^k-diffeomorphism."

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u is a C^k diffeomorphism if the following statements are true:

- u is of class C^k on V_1
- u is a bijection
- $u^{(-1)}$ is of class C^k on V_2

Exercise 3. Let

$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}, \quad (x,y) \longmapsto ye^{xy}.$$

$\partial_1 f: y^2e^{xy} = 1$

$\partial_2 f: e^{xy} + yxe^{xy}$

$\partial_{1,1} f: y^3e^{xy} = 1$

$\partial_{1,2} f = 3y^2e^{xy} + y^3xe^{xy} = 3$

$\partial_{2,2} f: xe^{xy} + xe^{xy} + yx^2e^{xy}$

$1 + (1 \quad 1)\begin{pmatrix} hx \\ hy \end{pmatrix} + \frac{1}{2}(hx \;\; hy)\begin{pmatrix} 1 & 3 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} hx \\ hy \end{pmatrix}$

1. Give the second order Taylor–Young expansion of f at $(0,1)$.

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Let $h = (hx, hy) \in \mathbb{R}^2$

$$f(hx, 1+hy) \underset{\|h\|\to 0}{=} 1 + (hx + hy) + \frac{1}{2}(hx + 3hy \;\; 3hx)\begin{pmatrix} hx \\ hy \end{pmatrix} + o(\|h\|^2)$$

$$= 1 + (hx + hy) + \frac{1}{2}(hx^2 + 3hyhx + 3hxhy) + o(\|h\|^2)$$

$$= 1 + hx + hy + \frac{hx^2}{2} + \cancel{3}hyhx \quad ?? \quad + o(\|h\|^2)$$

2. Deduce the value of the following limit (and briefly justify why this limit exists):

$$\ell = \lim_{(x,y)\to(0,1)} \frac{ye^{xy} - 1 - x - (y-1) - x^2/2}{x(y-1)}.$$

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