INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #5 — 12'

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Exercise 1. Let

$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}, \quad (x,y) \longmapsto xy - 2x - y.$$

Determine the directional derivative of f at $(1,1)$ in the direction $(1,2)$. No justifications required.

$\nabla_{(1,2)} f(1,1) = -1$ ✓

Exercise 2. Let $(E, \|\cdot\|_E)$ and $(F, \|\cdot\|_F)$ be two normed vector spaces, let U be an open subset of E, let $g : U \to F$ be a function, and let $q_0 \in U$.

1. Let $w \in E$. Recall the definition of the directional derivative of g at q_0 in the direction w (assuming that it exists):

$\nabla_w g(q_0) = \lim_{t \to 0} \dfrac{g(q_0 + t w) - g(q_0)}{t}$ ✓

2. We now assume that g is differentiable at q_0. We know that all the directional derivatives of g at q_0 exist. Give the relation between the directional derivatives of g at q_0 and the differential of g at q_0. No justifications required.

$\forall w \in E,\ \nabla_w g(q_0) = \cancel{D_{(q_0)} g}$

Exercise 3. Let f be the function defined by

$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}, \quad (y,x) \longmapsto ye^x + x.$$

There are no typos in the definition of f! Determine the first order partial derivatives of f at $(0,1)$ (you're given that they exist). No justifications required.

$\partial_1 f(0,1) = e^1 = e$ ✓

$\partial_2 f(0,1) = 0e^1 + 1 = 1$ ✓

$\partial_1 f(y,x) = e^x$

$\partial_2 f(y,x) = ye^x + 1$