INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #8 — 10'

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Exercise 1. Let U be an open subset of $\mathbb{R}^n$ (with $n \in \mathbb{N}^*$) and let $u : U \to \mathbb{R}$ be a function of class C^2. Let $q_0 \in U$. Recall the second-order Taylor–Young formula for u at q_0 (the general formula, with matrices).

let $h \in U$

$$u(q_0+h) \underset{\|h\|\to 0}{=} u(q_0) + J_{q_0}u \cdot [h]_{std} + \frac{1}{2}\, {}^t[h]_{std} \cdot H_{q_0}u \cdot [h]_{std} + o(\|h\|^2)$$

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Exercise 2. Let $n \in \mathbb{N}^*$ and let U_1 and U_2 be two open subsets of $\mathbb{R}^n$. Let $k \in \mathbb{N}^* \cup \{+\infty\}$ and let $\alpha : U_1 \to U_2$ be a function. Recall the definition of "α is a C^k-diffeomorphism."

α is C^k-diffeomorphism if:

- α is of class C^k on U_1
- α^{-1} is of class C^k on U_2
- α is a bijection

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$xe^{xy} + xe^{xy} + x^2ye^{xy}$

$\partial_1 f(x,y) = e^{xy} + xye^{xy}$

$\partial_2 f(x,y) = x^2e^{xy}$

$\partial^2_{1,1} f(x,y) = 2ye^{xy} + xy^2e^{xy}$

$\partial^2_{2,2} f(x,y) = x^3e^{xy}$

$\partial^2_{2,1} f(x,y) = 2xe^{xy} + x^2ye^{xy}$

Exercise 3. Let

$$f : \begin{array}{ccc} \mathbb{R}^2 & \longrightarrow & \mathbb{R} \\ (x,y) & \longmapsto & xe^{xy}. \end{array}$$

1. Give the Jacobian matrix and the Hessian matrix of f at $(1,0)$:

$J_{(1,0)}f = (1,\ 1)$ $\qquad H_{(1,0)}f = \begin{pmatrix} 0 & 2 \\ 2 & 1 \end{pmatrix}$

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2. Decude the second order Taylor–Young expansion of f at $(1,0)$.

let $h \in \mathbb{R}^2$ with $h = (h_x, h_y)$

$$f(h_x+1, h_y) \underset{(h_x,h_y)\to(0,0)}{=} 1 + h_x + h_y + 2h_xh_y + \frac{1}{2}h_y^2 + o(\|h\|^2)$$

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$f(1,0) = 1$

$(1,1)\begin{pmatrix} h_x \\ h_y \end{pmatrix}$

$\frac{1}{2}(h_x\ h_y)\begin{pmatrix} 0 & 1 \\ 1 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} h_x \\ h_y \end{pmatrix}$

$(h_y\ \ h_x + \frac{1}{2}h_y)\begin{pmatrix} h_x \\ h_y \end{pmatrix} = h_xh_y + h_yh_x + \frac{1}{2}h_y^2$