INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 2 — Quiz #9 — 10'

December 11, 2019

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Exercise 1. Recall the Global Inverse Function Theorem.

Let $f: U \to V$ be a bijection of class C^k $(k \geq 1)$ such that $\forall x \in U$, $D_x f$ is invertible, Then, f is a C^k diffeomorphism.

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Exercise 2. Let

$$\begin{aligned} \varphi : \quad \mathbb{R}^2 &\longrightarrow \mathbb{R}^2 \\ (x, y) &\longmapsto (-2x^2 + y^2, x^2 + y^2). \end{aligned}$$

You're given that φ is well-defined, of class C^∞, and you don't have to justify any of these properties.

1. Find the points $(x_0, y_0) \in \mathbb{R}^2$ around which we can apply the Local Inverse Function Theorem. No justifications required.

$J_{(x,y)}\varphi = \begin{pmatrix} -4x & 2y \\ 2x & 2y \end{pmatrix}$

$\det J_{(x,y)}\varphi = -4x \times 2y - 4xy$
$= -8xy - 4xy = -12xy.$

Now : $-12xy = 0 \iff x = 0$ or $y = 0$.

LIFT can be applied on :
$\{(x, y) \in \mathbb{R}^2 \mid x \neq 0, y \neq 0\} = \mathbb{R}^* \times \mathbb{R}^*$

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2. Give the Jacobian matrix of φ at $(1, 1)$.

$J_{(1,1)}\varphi = \begin{pmatrix} -4 & 2 \\ 2 & 2 \end{pmatrix}$

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3. On the back of this sheet are two pictures. The first one represents the domain of φ (i.e., $\mathbb{R}^2$) and the second one represents the codomain of φ (also $\mathbb{R}^2$).

On the first picture we have represented the point $A(1, 1)$ and a little house. In fact the house is very, very small, so the picture is not exactly to scale.

You're asked to determine the image of A by φ, and plot it on the second picture. You're also asked to plot the image of the little house by φ (still on the second picture). The overall scale of the image is not relevant. What's important, though, is to preserve the relative scale for the horizontal and vertical axes when plotting the image of the little house.

For your convenience, the necessary information is included on the back of this sheet.

$\varphi(1, 1) = (-2 + 1, 1 + 1)$
$= (-1, 2)$