INSA LYON

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Mechanics – Test 3

Personal formula sheet authorised (4 pages + 1page on usual links + one page on matrices of inertia) - Non programmable calculators are authorised

The system under consideration is a sander used by professionals which is equipped with a balancing ring to reduce vibrations. The objective of the study is to help design this balancing ring and establish a dynamic model in order to study vibrations (Figure 1).

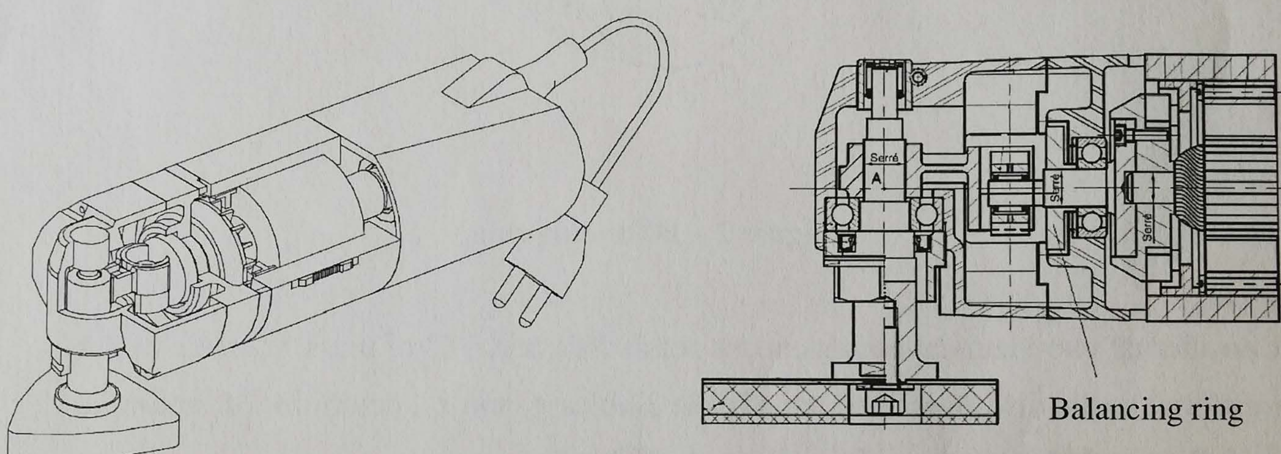

Figure 1 – Representation and technical drawing of the sander.

1. Design of the balancing ring

The balancing ring used to eliminate vibrations is tightly mounted on the rotor of the sander motor. The corresponding model is shown in Figure 2 which comprises :

- A homogeneous "disc" (cylinder of revolution of centre O_1) with two identical holes denoted t_1 and t_2. The mass of this disc with the two holes is M_t, its radius is R and its thickness is h. The axis of the disc $(O_1, \vec{x}_1)$.

- the two identical holes t_1 and t_2 are cylindrical of radius r_t and axes $(O_{t1}, \vec{x}_1)$ and $(O_{t2}, \vec{x}_2)$ respectively. The centres O_{t1} and O_{t2} of the two holes are located at a distance ℓ from O_1, centre of the disc, and shifted apart from axis $(O_1, \vec{z}_1)$ by an angle $\alpha \in \left]0, \pi/2\right[$

- An eccentric (homogeneous circular cylinder) made of the same material as the disc. Its mass is M_e, his radius r_e and its length h_e. Its axis $(O_{1*}, \vec{x}_1)$ is at distance e along $(O_1, \vec{z}_1)$ from the disc axis $(O_1, \vec{x}_1)$. The centre of the eccentric is denoted O_e.

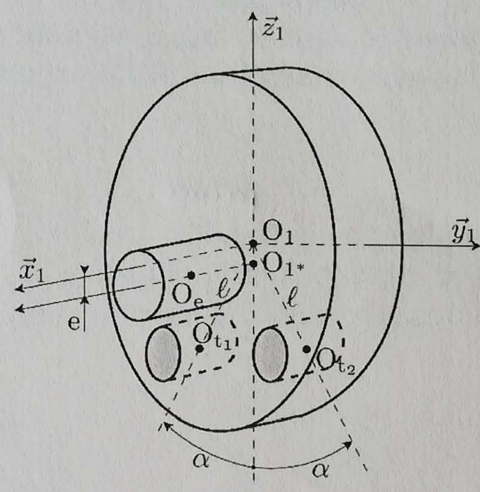

Figure 2 – Balancing ring.

Questions :

1.1. Considering two identical homogeneous solids ST1 and ST2 of mass m_t made of the same material as the disc and filling exactly the 2 holes t_1 and t_2, determine M_t and m_t in terms of R, r_t and M, the mass of the disc with NO holes.

1.2. Determine the position of the centre of inertia of the system (disc + holes +eccentric) in the coordinate system $(O_1, \vec{x}_1, \vec{y}_1, \vec{z}_1)$ in terms of the masses M_t, M_e, m_t and parameters h, h_e, e, ℓ and α.

1.3. What is the relationship between ℓ and α so that static balancing with respect to the axis of rotation $(O_1, \vec{x}_1)$ is achieved?

1.4. Give without any calculations the matrices of inertia of the two solids ST1 and ST2 at their centres O_{t1} and O_{t2} respectively in terms of m_t, r_t and h. For the rest of the problem, the elements of these matrices will be denoted $A_t, B_t, \ldots$ using Binet's notations.

1.5. Express the two matrices at point O_1 using the previous notations and m_t, ℓ and α.

1.6. Deduce the matrix of inertia of the disc with the two holes at point O_1 in terms of the parameters used in 1-5 and M, R and h.

1.7. Give **and justify** the a-priori form of the matrix of inertia of the whole system (disk + 2 holes +eccentric) at point O_1 without calculating its elements.

1.8. Is dynamic balancing of the assembly with respect to its axis of rotation $(O_1, \vec{x}_1)$ achieved? Justify.

2 – Vibration analysis

Model description:

The kinematic model shown in Figure 3 comprises:
– ground S_0 with the coordinate system $(O,\vec{x}_0,\vec{y}_0,\vec{z}_0)$ corresponding to the sander casing held by the operator and considered here as fixed.
– the driving shaft with the eccentric S_1 connected to the ground by a revolute joint of axis $(O,\vec{x}_{1,0})$ and parameter $\psi_1=(\vec{z}_0,\vec{z}_1)$
– the operating part of the sander S_2 connected to the ground by a cylindrical joint of axis $(O,\vec{z}_{2,0})$ and parameters $\psi_2=(\vec{x}_0,\vec{x}_2)$ and $z=\overrightarrow{OP}.\vec{z}_{0,2}$

The link with no parameter between S_1 and S_2 is simulated as a permanent point contact at A_1 attached to S_1 so that A_1 is permanently lying in the plane $(O,\vec{x}_2,\vec{z}_{0,2})$ fixed to S_2. One gives $\overrightarrow{OA_1}=-L\,\vec{x}_{0,1}-e\,\vec{z}_1$.

Mass geometry:

S_1 : mass M_1, its centre of inertia is on the $(O,\vec{x}_{1,0})$ axis, matrix of inertia at point O:

$$[I]_{O,S_1}=\begin{bmatrix} A_1 & 0 & -E_1 \\ 0 & B_1 & 0 \\ -E_1 & 0 & C_1 \end{bmatrix}_{R_1}$$

S_2 : mass M_2, its centre of inertia G_2 is defined by $\overrightarrow{PG_2}=-a\,\vec{x}_2+b\,\vec{z}_{0,2}$, matrix of inertia at point G_2 : $[I]_{G_2,S_2}=\begin{bmatrix} A_2 & 0 & -E_2 \\ 0 & B_2 & 0 \\ -E_2 & 0 & C_2 \end{bmatrix}_{R_2}$

Hypotheses:

H1: $(O,\vec{x}_0,\vec{y}_0,\vec{z}_0)$ is considered as Galilean (Newtonian)
H2: All the joints/links are supposed to be perfect.
H3: $\vec{z}_0$ is the upward vertical axis
H4: a massless linear spring of stiffness k and length at rest ℓ_0 is mounted between point O of S_0 and point P of S_2 (this spring simulates the elasticity of the joint S_0/S_2 and sander head).
H5: The sander head is supposed to be in contact with a planar horizontal surface (P fixed to S_2 lies on this plane). Consequently and by account on symmetry, the friction forces cancel two by two but generate a pitching torque so that the contact force wrench reduces to:

$$\{F_{0/2}\}\begin{cases} \vec{R}_{0/2} = Z_{02}\,\vec{z}_{0,2} \\ \vec{M}_{0/2}(P) = N_{02}\,\vec{z}_{0,2} \end{cases}$$

where Z_{02} is supposed to be **known** (imposed by the operator) and N_{02} is **unknown**, the pitching parameter f_p is given.

H6: an electric motor delivers a known driving torque $C_{m/1}\,\vec{x}_{0,1}$ on the shaft of S_1.

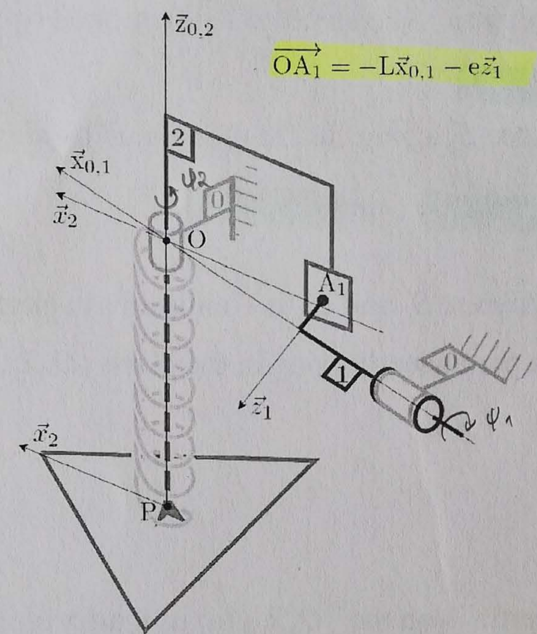

Figure 3 – Model of sander (with spring)

Questions:

2-1 Develop the constraint equation associated with the condition that point A_1 always lies in the plane $(O, \vec{x}_2, \vec{z}_{0,2})$. Show that it leads to the equation $\tan\psi_2 = -\frac{e}{\ell}\sin\psi_1$.

2-2 Give the a-priori form of contact force wrench for link S_2 / S_1 (expressed in (R_2)).

2-3 Give the wrench (sum and moment) of the actions exerted by the spring on S_2.

2-4 Calculate the Galilean kinetic wrench (sum and moment) of S_1 at point O.

2-5 Calculate the Galilean kinetic wrench (sum and moment) of S_2 at point O.

2-6 Calculate the Galilean dynamic wrench (sum and moment) of S_1 at point O.

2-7 Based on the previous calculations, what is the best strategy to calculate the Galilean dynamic moment of S_2 at point O *(Please, do not develop the calculations, just give the methodology!)*? Justify.

2-8 Give a complete analysis of the system (unknowns/equations).

2-9 Deduce the minimum system equations.

2-10 Develop the dynamic moment equation applied to S_1 at point O projected in the $\vec{x}_{0,1}$ direction.

2-11 Is this equation an equation of motion? Please justify.

2-12 Develop the law of Coulomb at the interface between the sander head and the surface.