MECHANICS – test 1: Correction

Duration 1h30 **24/11/2016**

A. Static analysis

1. Equilibrium of the container

The static equilibrium of the set (containers M + 4 cables) gives: :

$\boxed{\overrightarrow{F_{C/M}} + \overrightarrow{P_M} = \vec{0}}$

One can then deduce $\boxed{\overrightarrow{F_{C/M}} = Mg\overrightarrow{z_0}}$

The container M is subjected to :

- The gravity $\overrightarrow{P_M} = -Mg\overrightarrow{z_{0,M}}$,
- The actions of the 4 cables $\overrightarrow{F_{C1/M}}, \overrightarrow{F_{C2/M}}, \overrightarrow{F_{C3/M}}, \overrightarrow{F_{C4/M}}$

The actions of the cables can be expressed in $(\vec{x}_M, \vec{y}_M, \vec{z}_M)$ as follows:

$$\overrightarrow{F_{C1/M}} = {}_M\begin{vmatrix} F\cos\beta\sin\gamma \\ F\cos\beta\cos\gamma \\ F\sin\beta \end{vmatrix}, \overrightarrow{F_{C2/M}} = {}_M\begin{vmatrix} -F\cos\beta\sin\gamma \\ F\cos\beta\cos\gamma \\ F\sin\beta \end{vmatrix}, \overrightarrow{F_{C3/M}} = {}_M\begin{vmatrix} -F\cos\beta\sin\gamma \\ -F\cos\beta\cos\gamma \\ F\sin\beta \end{vmatrix}, \overrightarrow{F_{C4/M}} = {}_M\begin{vmatrix} F\cos\beta\sin\gamma \\ -F\cos\beta\cos\gamma \\ F\sin\beta \end{vmatrix}$$

The fundamental principle of statics permits writing in $(\vec{x}_M, \vec{y}_M, \vec{z}_M)$:

$$\overrightarrow{F_{C1/M}} + \overrightarrow{F_{C2/M}} + \overrightarrow{F_{C3/M}} + \overrightarrow{F_{C4/M}} + \overrightarrow{P_M} = \vec{0}$$

Which leads to $\boxed{F = {}^{\mathrm{Mg}}/_{4\sin\beta}}$

1. Pulley P2, The system is supposed to remain in plane $(O_0, \vec{y}_1, \vec{z}_{0,1})$, so that $\alpha = 0$.

a. Determine the force wrench at point D resulting from tensions $\overrightarrow{T_1} = -T_1\overrightarrow{y_2}$ and $\overrightarrow{T_2} = -T_2\overrightarrow{z_0}$ in the two strands of cable (Figure 2).

By projecting $\overrightarrow{T_1} = -T_1\overrightarrow{y_2}$ in basis (0), one obtains : $\overrightarrow{T_1} = {}_{0,1}\begin{vmatrix} 0 \\ -T_1\cos\theta \\ -T_1\sin\theta \end{vmatrix}$, the equivalent wrench is :

$$\overrightarrow{T_{1+2/P2}} = {}_{0,1}\begin{vmatrix} 0 \\ -T_1\cos\theta \\ -T_1\sin\theta - T_2 \end{vmatrix}$$

$$\overrightarrow{M_{1+2/P2}(D)} = \overrightarrow{DJ} \wedge \overrightarrow{T_1} + \overrightarrow{DI} \wedge \overrightarrow{T_2} = r\overrightarrow{z_2} \wedge \overrightarrow{T_1} + r\overrightarrow{y_0} \wedge \overrightarrow{T_2} = r(T_1 - T_2)\overrightarrow{x_{0,2}}$$

b. From the equilibrium of the pulley P2, find the relation between $\overrightarrow{T_1}$ and $\overrightarrow{T_2}$, express these two forces in terms of M and g.

In addition to the cable action, pulley P2 is connected to the boom (2) by a revolute joint of axis (D, $\overrightarrow{x_{1,2}}$) , the corresponding joint is:

$$\overrightarrow{R_{2/P2}} = {}_{0,1}\begin{vmatrix} X_{2P2} \\ Y_{2P2} \\ Z_{2P2} \end{vmatrix}$$

$$\overrightarrow{M_{2/P2}(D)} = {}_{0,1}\begin{vmatrix} 0 \\ M_{2P2} \\ N_{2P2} \end{vmatrix}$$

The static moment equilibrium at point D of pulley P2 gives $\boxed{r(T_1 - T_2) = 0}$ and then $\boxed{T_1 = T_2 = Mg}$

a. Derive the force wrench (sum and moment) exerted by boom (2) on pulley P2 at point D

$$\overrightarrow{R_{2/P2}} = \begin{vmatrix} 0 \\ Y_{2P2} = Mg\cos\theta \\ Z_{2P2} = Mg\,(\sin\theta + 1) \end{vmatrix}_{0,1} \quad \overrightarrow{M_{2/P2}(D)} = \vec{0}$$

2. Equilibrium of boom (2)

a. Give the list of all the external mechanical actions on boom (2).

The boom (2) is subjected to :

- The gravity : $\overrightarrow{P_2} = -m_2 g\overrightarrow{z_{0,1}}$
- The pulley P_2 action at D, $\overrightarrow{R_{P2/2}} = \begin{vmatrix} 0 \\ -Y_{2P2} = -Mg\cos\theta \\ -Z_{2P2} = -Mg\,(\sin\theta + 1) \end{vmatrix}_{0,1}$
- Actions of solid (1) at A, revolute joint of axis (A, $\overrightarrow{x_{1,2}}$) $\overrightarrow{R_{1/2}} = \begin{vmatrix} X_{12} \\ Y_{12} \\ Z_{12} \end{vmatrix}_{0,1}$
- Actions of solid (1) at C, revolute joint of axis (C, $\overrightarrow{x_{1,2}}$) $\overrightarrow{R_{4/2}} = V.\overrightarrow{y_{3,4}} = \begin{vmatrix} X_{42} = 0 \\ Y_{42} = V\cos\psi \\ Z_{42} = V\sin\psi \end{vmatrix}_{0,1}$

b. Develop the static equilibrium equations for the boom and determine the force wrenches in joints (1-2) and (4-2) in terms of masses M and m_2 (along with geometrical parameters).

$$\overrightarrow{M(\overrightarrow{P_2}, A)} = \overrightarrow{AG} \wedge \overrightarrow{P_2} = \frac{l}{2}\overrightarrow{y_2} \wedge \overrightarrow{P_2} = \begin{vmatrix} 0 \\ l/2\cos\theta \\ l/2\sin\theta \end{vmatrix}_{0,1} \wedge -m_2 g\overrightarrow{z_{0,1}} = -m_2 g\, l/2\cos\theta\,\overrightarrow{x_{0,2}}$$

$$\overrightarrow{M(\overrightarrow{R_{P2/2}}, A)} = \overrightarrow{AD} \wedge \overrightarrow{R_{P2/2}} = l\overrightarrow{y_2} \wedge \overrightarrow{R_{P2/2}} = \begin{vmatrix} 0 \\ l\cos\theta \\ l\sin\theta \end{vmatrix}_{0,1} \wedge \begin{vmatrix} 0 \\ -Mg\cos\theta \\ -Mg\,(\sin\theta + 1) \end{vmatrix}_{0,1} = -Mgl\cos\theta\,\overrightarrow{x_{0,2}}$$

$$\overrightarrow{M(\overrightarrow{R_{4/2}}, A)} = \overrightarrow{AC} \wedge \overrightarrow{R_{4/2}} = \begin{vmatrix} 0 \\ c\cos\theta + e\sin\theta \\ c\sin\theta - e\cos\theta \end{vmatrix}_{0} \wedge \begin{vmatrix} 0 \\ V\cos\psi \\ V\sin\psi \end{vmatrix}_{0,1} = V(c\sin(\psi - \theta) + e\cos(\psi - \theta))\overrightarrow{x_{0,2}}$$

The moment equilibrium at A gives:

$$0 = -m_2 g\, l/2\cos\theta - Mgl\cos\theta + V(c\sin(\theta - \psi) + e\cos(\theta - \psi))$$

Then, we deduce $V = \frac{(Ml + m_2 l/2)g\cos\theta}{(c\sin(\theta-\psi) + e\cos(\theta-\psi))}$

The force equilibrium of solid (2) permits determining Y_{12} and Z_{12}:

$$\overrightarrow{R_{1/2}} = \begin{vmatrix} 0 \\ Y_{12} = Mg\cos\theta - V\cos\psi \\ Z_{12} = m_2 g + Mg\,(\sin\theta + 1) - V\sin\psi \end{vmatrix}_{0,1}$$

4. Graphical statics

In figure $\overrightarrow{R_{P2/2}}$ results from the vectorial sum of $\overrightarrow{T_1}$ and $\overrightarrow{T_2}$.
The set (3-4) is subjected to two sliding vectors of support (BC), hence $\overrightarrow{R_{4/2}}$ is along (BC).
The boom (2) is subjected to 3 actions at A, C and D. The support lines of these three actions cross at point K, which permits determining the of $\overrightarrow{R_{1/2}}$.

Then , graphically the triangle of the forces resulting from the force equilibrium of the boom is constructed :
$\overrightarrow{R_{P2/2}} + \overrightarrow{R_{4/2}} + \overrightarrow{R_{1/2}} = \vec{0}$

In the figure, one measures $\|\overrightarrow{F_{4/2}}\| = 23.7\ cm$, and $\|\overrightarrow{T_2}\| = 3.5\ cm = 45.10^3 * 9.81N$

$$\|\overrightarrow{F_{4/2}}\| = 23.7\ cm = 2994kN$$

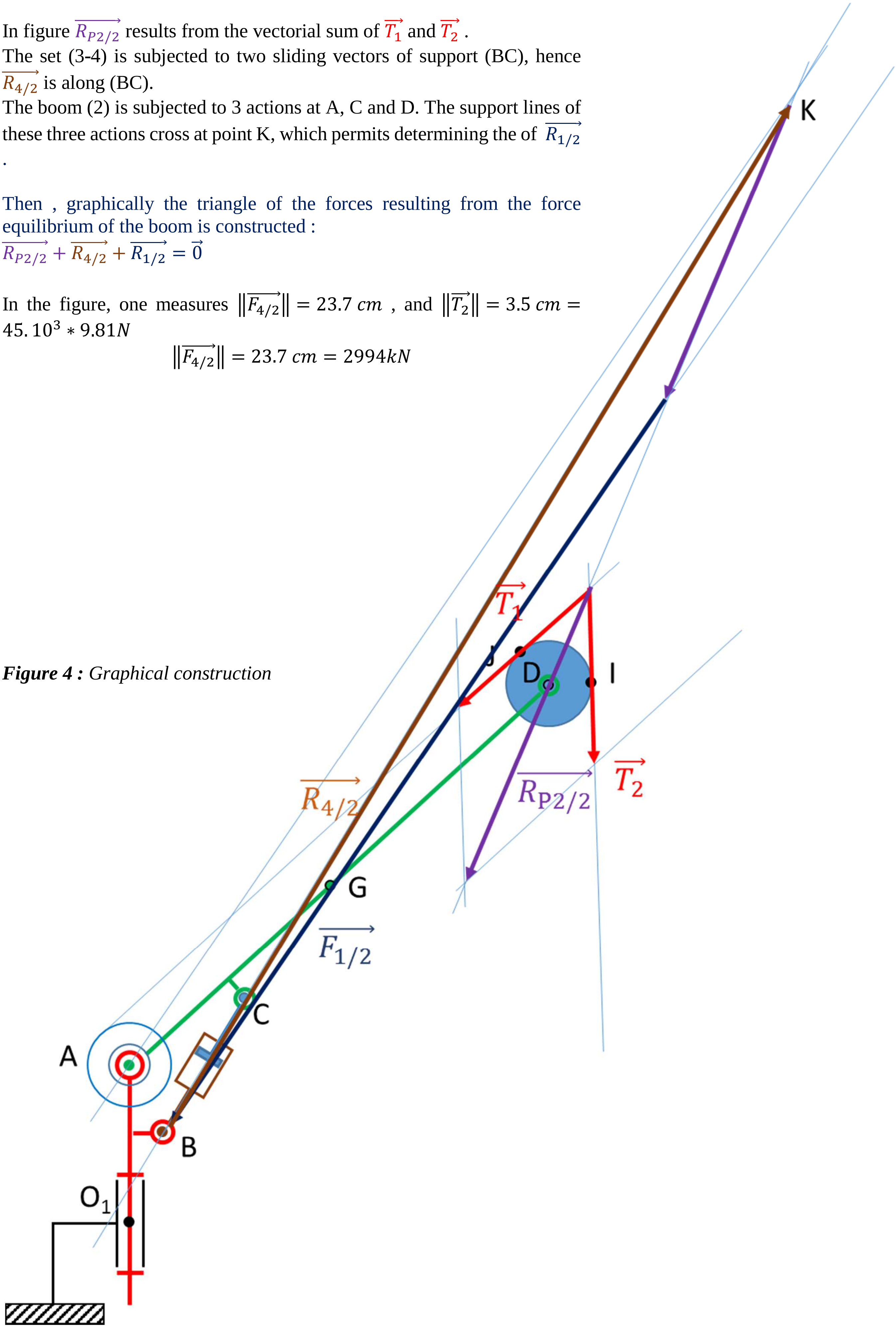
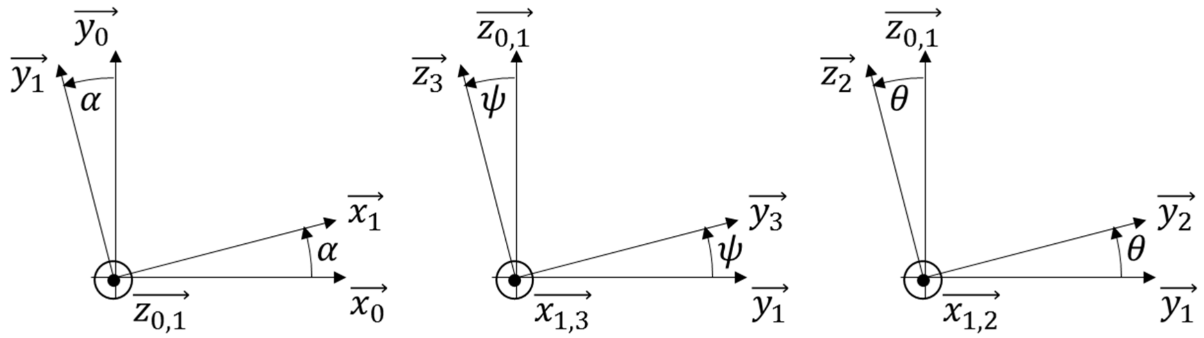


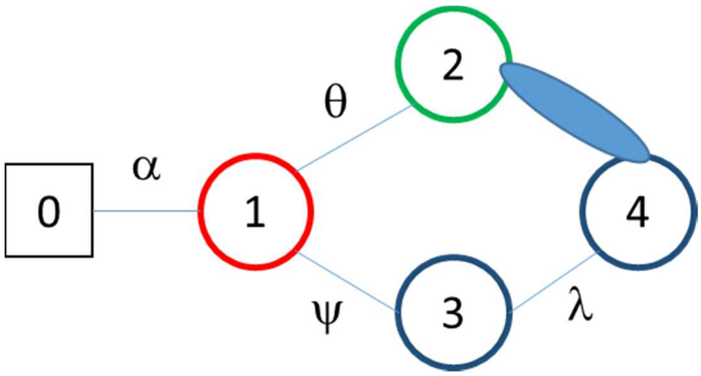
Figure 4 : *Graphical construction*

<u>**B. Kinematics**</u>

1. Graph of links and change of basis diagrams

Here the pulleys are not considered.





1. Constraint equation(s) and degree of mobility

a. Give the constraint equation generated by the revolute joint at C (vector form)

The revolute joint at C between (4) and (2) imposes $\boxed{\overrightarrow{C_4C_2} = \vec{0}}$

b. Develop the constraint equation(s)

$$\boxed{\overrightarrow{C_4C_2} = \vec{0} = \overrightarrow{C_4B} + \overrightarrow{BO_1} + \overrightarrow{O_1A} + \overrightarrow{AC_2}}$$

$$\boxed{\overrightarrow{C_4C_2} = -\lambda\overrightarrow{y_3} + {}_1\begin{vmatrix}0\\-a\\-b\end{vmatrix} + {}_1\begin{vmatrix}0\\0\\k\end{vmatrix} + {}_2\begin{vmatrix}0\\c\\-e\end{vmatrix}}$$

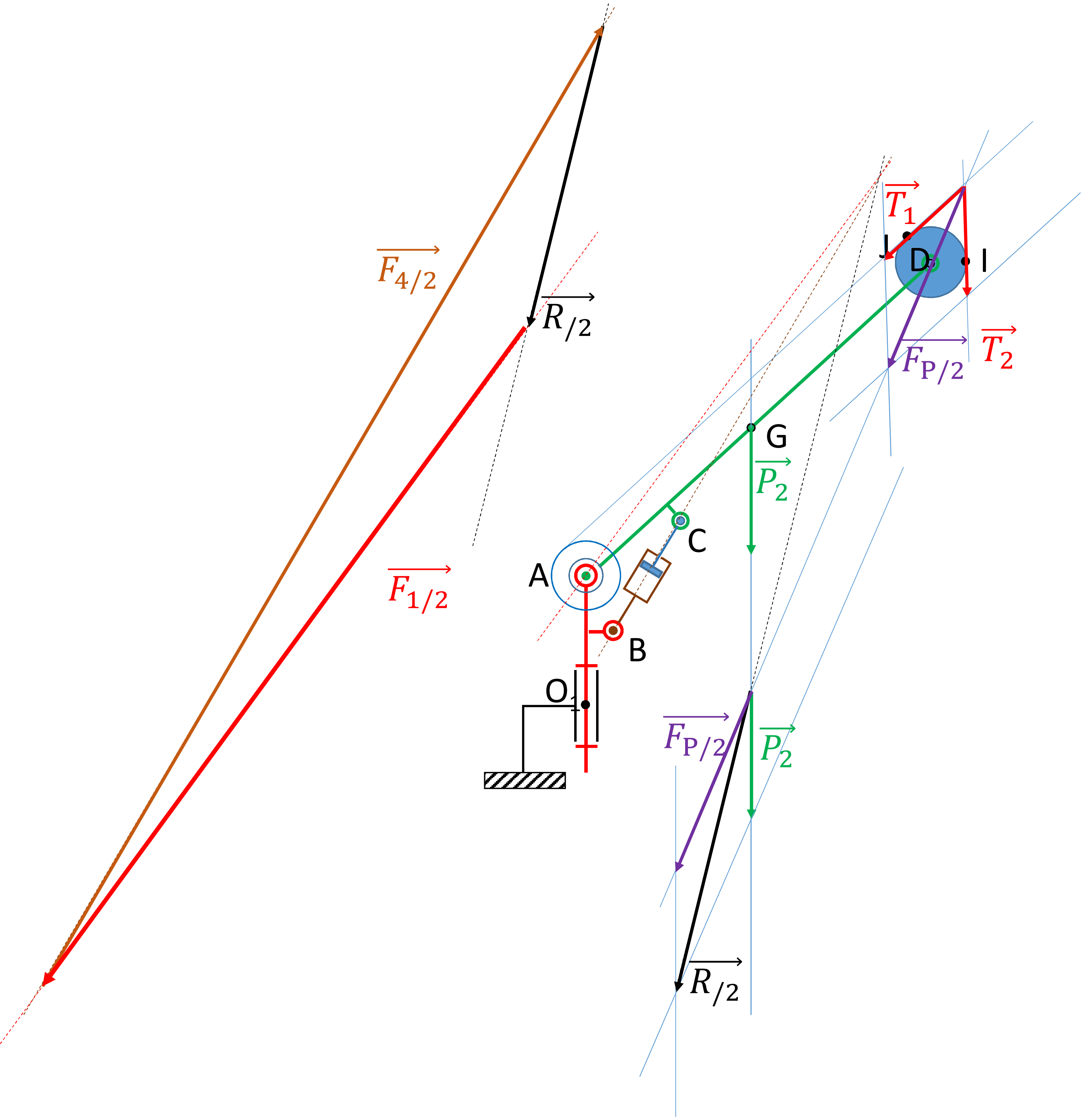
$$\left|\begin{array}{l}-\lambda\cos\psi - a + c\cos\theta + e\sin\theta = 0\\ -\lambda\sin\psi - b + k + c\sin\theta - e\cos\theta = 0\end{array}\right.$$

c. Determine the degree of mobility of the system.

- 4 kinematics parameters
- 2 equations

m= 4-2=2.

For controlling the crane motions, one needs to control the revolute joint 0-1 and the hydraulic jack extension (3-4).



$\overrightarrow{F_{4/2}}$
$\overrightarrow{R_{/2}}$
$\overrightarrow{F_{1/2}}$
$\overrightarrow{T_1}$
J
D
I
$\overrightarrow{F_{P/2}}$
$\overrightarrow{T_2}$
G
$\overrightarrow{P_2}$
C
A
B
O
$\overrightarrow{F_{P/2}}$
$\overrightarrow{P_2}$
$\overrightarrow{R_{/2}}$