INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

1^{st} cycle - 2^{nd} year - Scan section

MECHANICS - TEST N° 5

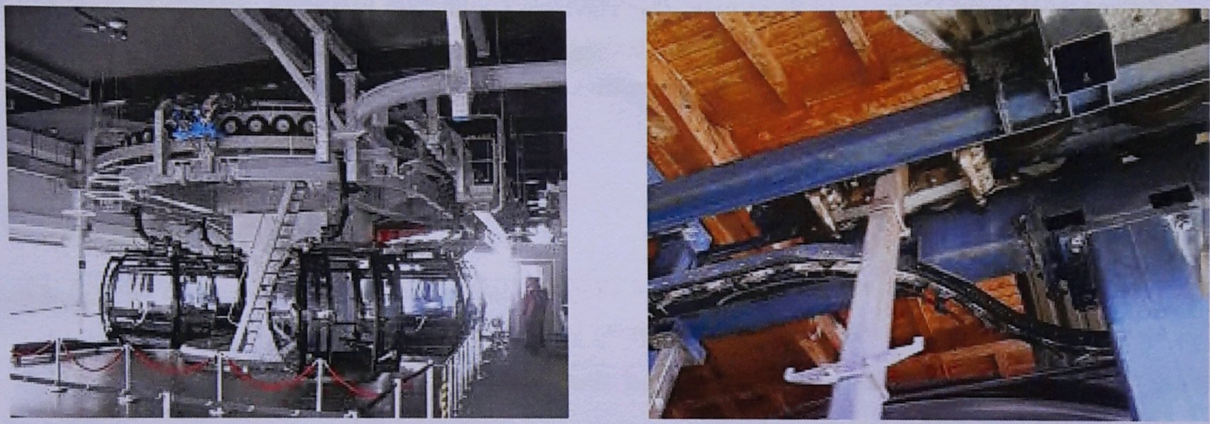
11/06/2018 – 3H00 (14H00-17H00)

Authorized documents : Formula sheet (4 pages + table of usual joints + table of inertia matrices)
non programmable calculator

Marking scheme – Part 1 : 3pts; Part 2 : 7pts; Part 3 : 2pts; Part 4 : 2,5pts; Part 5 : 5,5pts.
Parts 1, 2, 3 and 5 are independent. In Part 4, questions 4.1 and 4.2 depend on parts 1 and 2

Some elements of a cabin ski lift dynamics (cable car, when passing in the station)

The system studied in this test consists in a cabin ski lift (or gondola) when passing in the ski lift station for loading and unloading passengers (figure 1), during the entrance and exit phases from the station. The present study aims at establishing a dynamic model of the cabin ski lift in order to help its design and analyse its dynamic behaviour.



a) b)

FIGURE 1 – a - cabin ski lift when travelling through the station; b - chariot to which the cabin is suspended

Model description

When travelling through the station for loading or unloading skiers, the cabin is suspended to a rail via a chariot equipped with rollers (that roll on the rail), (figure 1-b). The displacement of the chariot is controlled by driving wheels in contact with a pad linked to the chariot(figure 1-a.). For the sake of simplicity, it is considered that the pad is in contact with only one wheel .

A planar system is assumed for the present analysis. It is composed of the following solids (figure 2).

- Rail S_0 is considered as the reference solid, it is associated with frame $R_0=(O,\vec{x}_0,\vec{y}_0,\vec{z}_0)$ where O is the driving wheel center, $\vec{z}_0$ the upward vertical axis and $\vec{y}_0$ along the rail (supposed to be horizontal).
- Chariot S_1 equipped with rollers (not represented on the kinematic scheme) rolling on the rail. These rollers permit the displacement of the chariot along the rail axis $\vec{y}_{0,1}$ so that the link between S_0 and S_1 is assimilated to a prismatic joint of axis $\vec{y}_{0,1}$. The frame linked to the chariot is $R_1=(O_1,\vec{x}_{0,1},\vec{y}_{0,1},\vec{z}_{0,1})$ with O_1 such that its vertical distance to O equals H.
  Parameter of motion S_1/S_0 : $y = \overrightarrow{OO_1} \cdot \vec{y}_{0,1}$.
- Pad S_2, of height d, and physical frame $R_2=(O_2,\vec{x}_{0,2},\vec{y}_{0,2},\vec{z}_{0,2})$, is linked to chariot S_1 by a prismatic joint of axis $\vec{z}_{1,2}$.
  Parameter of motion S_2/S_1 : $z = \overrightarrow{O_1O_2} \cdot \vec{z}_{1,2}$.
- Cabin S_3, of frame $R_3=(O_3,\vec{x}_{0,3},\vec{y}_3,\vec{z}_3)$, is linked to S_2 by a revolute joint of axis $(O_3,\vec{x}_{2,3})$.

$\vec{z}_{0,1,2,3,4}$, $\vec{y}_{0,1,2}$,

Parameter of motion S_3/S_2 : $\psi = (\vec{y}_2, \vec{y}_3)$.

— The driving wheel S_4, of radius R, and frame R_4=(O,$\vec{x}_{0,4}$,$\vec{y}_4$,$\vec{z}_4$), is linked to S_0 by a revolute joint of axis (O,$\vec{x}_{0,4}$).

Parameter of motion S_4/S_0 : $\theta = (\vec{y}_0, \vec{y}_4)$.

Moreover, wheel S_4 is in **contact at I** with the pad S_2 where **no slipping** is assumed. The geometric constant parameters are given in figure 2.

Hypotheses :

H_1 : A planar model is considered.

H_2 : $\vec{z}_0$ is the upward vertical axis and g the acceleration of the gravity field

H_3 : the rail is linear.

H_4 : All joints are perfect except the contact between wheel S_4 and pad S_2 where, at point I, there is friction and a resistant rolling moment modelled by a Coulomb model (friction coefficient f and rolling parameter h).

H_5 : The actions of the rollers transmitted by the chariot on the pad are modelled by a linear spring between chariot S_1 at point O_1 and pad S_2 at point O_2. The spring stiffness is k, its free length is ℓ_0.

H_6 : A motor mounted between S_0 and wheel S_4 exerts a driving torque $C_m\vec{x}_{0,4}$ on the wheel .

H_7 : It is assumed that the cabins move in the positive direction along $\vec{y}_0$ and therefore that **$\dot{\theta}$ is always positive.**

Mass and inertia characteristics :

S_1 : Mass m_1.

S_2 : Mass m_2.

S_3 : Mass m_3, inertia centre G_3 such that $\overrightarrow{O_3G_3} = -a\vec{z}_3$, the inertia moment with respect to (G_3,$\vec{x}_3$) is I_3.

S_4 : inertia centre O, the inertia moment with respect to (O,$\vec{x}_4$) is I_4.

IMPORTANT : the justifications and the writing quality will be considered in the marking

Part 1 : Kinematic analysis

The objective of this part is to develop the constraint equations.

Questions :

1.1. Plot the graphs of links.

1.2. Determine and develop the constraint equation(s) associated with the contact at I.

1.3. Determine and develop the constraint equation(s) associated with the no slipping condition at I.

Part 2 : Dynamic analysis

The objectives of this part are, (a) to determine the equations of motion for the cabin, and (b) to express the force wrench components of the joint between S_2 and S_3.

Questions :

2.1. Balance of the problem formulation in order to obtain the equation of motions - Define the minimum system equations.

2.2. Develop the equations derived from the general theorems of dynamics (it is considered that z = constant).

2.3. Express the Coulomb friction law at point I for the resisting rolling torque.

2.4. Express the equations of motion by eliminating y and its time derivatives (it is considered that $y = R\theta + \text{constant}$).

2.5. Give and develop the general theorems (vectorial form, no projection) which are needed to obtain the components of the joint wrench between S_2 and S_3.
***Indication** :express the wrench components in R_3.*

Part 3 : Control of the driving wheel

The motor is controlled such that the chariot (and therefore the pad) has a given constant acceleration A in the positive direction of axis $\vec{y_0}$. By assuming that the cabin is not oscillating ($\dot{\psi} = 0$) and that the weight action is negligible compared to that of the spring, the normal contact force from the pad on the wheel can be written as $Z_{2/4} = -\mathrm{k}(\mathrm{H} - \mathrm{R} - \ell_0)$. Moreover, by neglecting the resisting rolling moment, the rotation of the wheel is governed by the equation $\mathrm{I}_4\ddot{\theta} = \mathrm{R}Y_{2/4} + C_\mathrm{m}$. As previously $y = \mathrm{R}\theta + \mathrm{c}^{\mathrm{ste}}$ and the contact tangent force of the pad on the wheel is resisting to the wheel rotation ($Y_{2/4} < 0$).

Questions :

3.1. What is the condition on the driving torque so that there is no slipping between the wheel and the pad (this condition must be expressed in terms of the constant parameters of the model and the given acceleration A) ?

3.2. If this condition is not satisfied, on which parameter(s) is it possible to act for correcting that (without changing A) ? Comment.

Part 4 : Cabin dynamic behaviour analysis

By considering the equations of part 2, the dynamics of the cabin is now studied in the vicinity of stationary positions such that :

- $\dot{\theta} = \omega = \mathrm{c}^{\mathrm{st}}$ with ω given,
- $\dot{y} = \dot{y}^* = \mathrm{c}^{\mathrm{st}}$,
- $z = z^* = \mathrm{c}^{\mathrm{st}}$,
- $\psi = \psi^* = \mathrm{c}^{\mathrm{st}}$.

Questions :

4.1. Determine the stationary positions ($\dot{y}^*$, z^* et ψ^*) of the system.

4.2. For these stationary positions, determine the contact force wrench at point I and the torque C^*_m driving wheel S_4.

The equation related to the motion of the cabin can be written as $(\mathrm{I}_3 + \mathrm{m}_3\mathrm{a}^2)\,\ddot{\psi} + \mathrm{m}_3\mathrm{a}\cos\psi\ddot{y} = -\mathrm{m}_3\mathrm{g}\mathrm{a}\sin\psi$. It is assumed that the wheel rotates at constant speed $\dot{\theta} = \omega$ and therefore the chariot moves also at constant velocity $\dot{y} = \mathrm{R}\omega$.

For the analysis of small motions in the vicinity of the stationary position $\psi = 0$, one sets :

- $\psi = \overline{\psi}$,
- $\dot{\psi} = \dot{\overline{\psi}}$,
- $\ddot{\psi} = \ddot{\overline{\psi}}$,

where $\overline{\psi}$, $\dot{\overline{\psi}}$ and $\ddot{\overline{\psi}}$ are very small first order terms.

Questions :

4.3. Give the equation for the small oscillations of the cabin around the stationary position $\psi = 0$.

4.4. Determine the solution $\overline{\psi}(t)$ of this equation and discuss the stability of the stationary position.

4.5. On which parameters and in what direction should one act to decrease the oscillation frequency ?

Part 5 : Cabin dynamics when entering the station

When entering the station, the speed reduction is insured by the first wheels acting as brake motor. This part aims at determining the brake torque C_{mf} of the wheel motor for obtaining a given deceleration A<0 and slipping ($\vec{V}\,(\mathrm{I}, 4/2) \neq \vec{0}$) between the wheels and the pad. To do this, one will use the kinetic energy theorem applied to the set of solids {chariot + pad + cabin + wheel} and the Coulomb friction law. In this entrance of station phase, the parameters are now defined with respect to R_{0^*} defined by the constant inclination angle $\alpha = (\vec{y}_0, \vec{y}_{0^*})$ of the rail with respect to R_0 (figure 3). In this section, one does not consider small motions any longer. Also z is constant.

{1+2+3+4} -

Questions :

5.1. Calculate the Galilean kinetic energy of $\{S_1 + S_2 + S_3 + S_4\}$.

5.2. Calculate the Galilean powers of all the mechanical actions.

5.3. Apply the kinetic energy theorem.

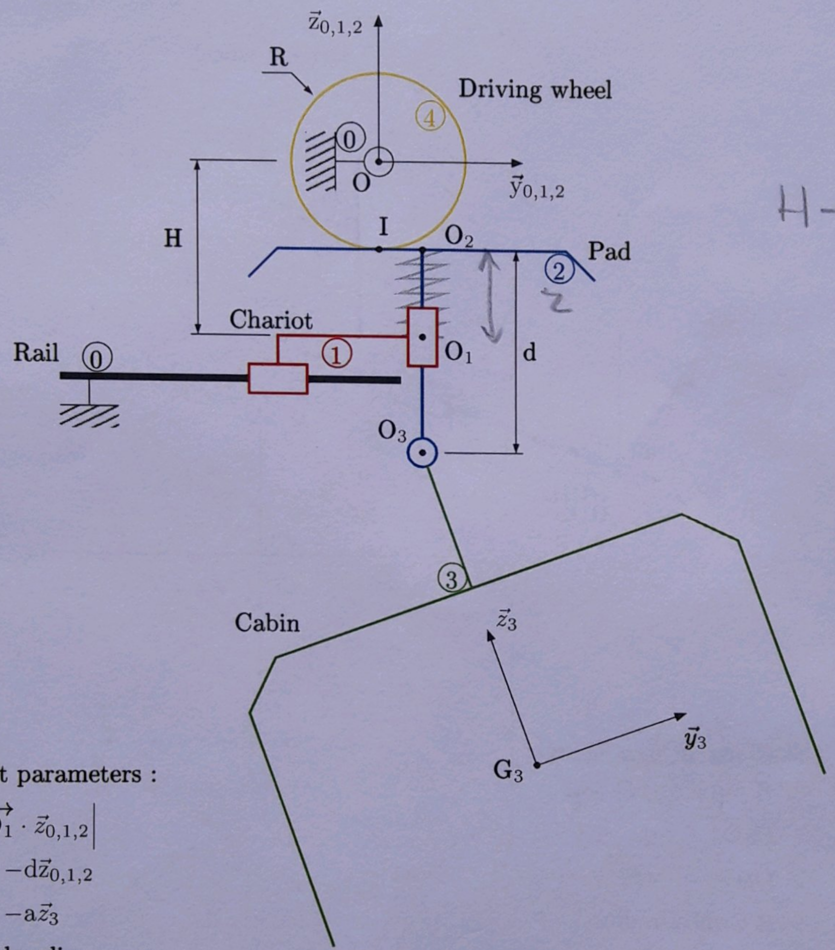
One considers a slip rate of 10% ($\frac{\dot{y}-\mathrm{R}\dot{\theta}}{\dot{y}} = 0.1$). It is also supposed that the cabin remains in a vertical position ($\dot{\psi} = 0$) and that the rolling resistance is negligible.

5.4. By using the Coulomb friction law and simplifying the equation obtained in the previous question, express the braking torque C_{mf} in terms of the normal force from the wheel on the pad and the deceleration A, among others parameters.

5.5. Which general theorem should one apply in order to determine the normal force exerted by the wheel on the pad (do not develop this equation) ?

$$P^{\circ}_{contact} = \left\{ \begin{matrix} [0 \,,\, Y_{24} \,,\, Z_{24}]_{0^*} \\ \overrightarrow{R^2_4}(I) \end{matrix} \right. \otimes \left\{ \begin{matrix} \overrightarrow{\Omega^2_4} = \dot{\theta}\,\vec{x} \\ \overrightarrow{V^2_4}(I) \neq \vec{O} = \lambda \overrightarrow{y_{0^*}} + \mu \overrightarrow{z_{0^*}} \end{matrix} \right.$$

$$= Y_{24}\,\lambda\,\overrightarrow{y_{0^*}} + Z_{24}\,\mu\,\overrightarrow{z_{0^*}} + \|\overrightarrow{R^2_4}\| \cdot \dot{\theta}$$



$H - z = R$

Constant parameters :

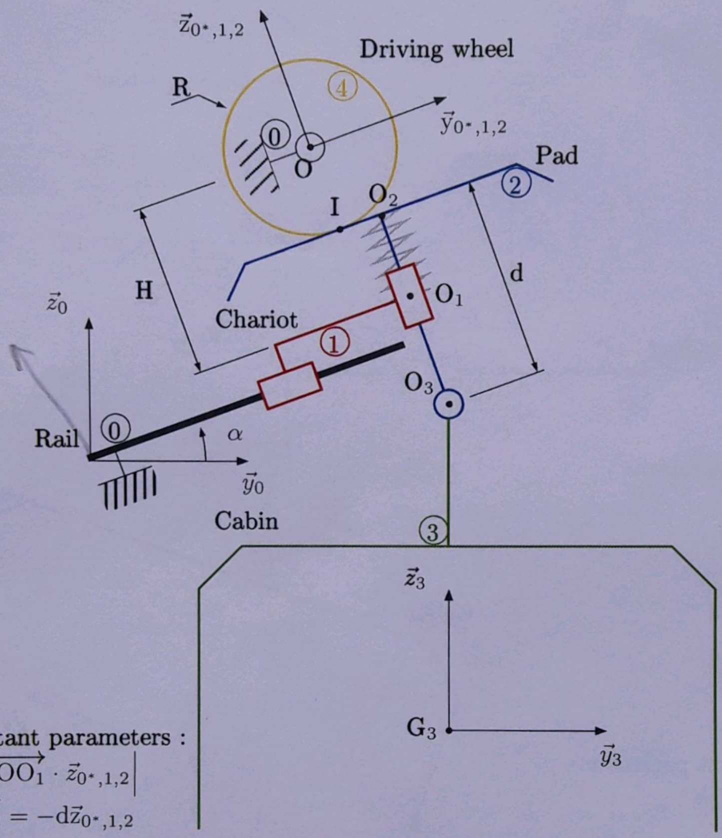
$\mathrm{H} = \left|\overrightarrow{OO_1} \cdot \vec{z}_{0,1,2}\right|$

$\overrightarrow{O_2O_3} = -d\vec{z}_{0,1,2}$

$\overrightarrow{O_3G_3} = -a\vec{z}_3$

R : wheel radius

FIGURE 2 – Kinematic scheme of the system during the exit phase from the station **(for the first 4 parts)**



Constant parameters :
$H = \left|\overrightarrow{OO_1} \cdot \vec{z}_{0^*,1,2}\right|$
$\overrightarrow{O_2O_3} = -d\vec{z}_{0^*,1,2}$
$\overrightarrow{O_3G_3} = -a\vec{z}_3$
R : wheel radius
$\alpha = (\vec{y}_0, \vec{y}_{0^*})$

FIGURE 3 – Kinematic scheme of the system during the entrance phase in the station **(only for part 5)**.

$\overrightarrow{y_{0*}} = \cos\alpha\,\overrightarrow{y_0} + \sin\alpha\,\overrightarrow{z_0}$

$\overrightarrow{z_{0*}} = -\sin\alpha\,\overrightarrow{y_0} + \cos\alpha\,\overrightarrow{z_0}$

$\overrightarrow{y_3} = +\cos\psi\,\overrightarrow{y_{0*}} + \sin\psi\,\overrightarrow{z_{0*}}$

$= \left[\cos\psi\cos\alpha + \sin\psi(-\sin(\alpha))\right]\overrightarrow{y_0} + \left[\cos\alpha\sin\psi + \cos\psi\sin\alpha\right]\overrightarrow{z_0}$