**A one-page A4 personal formula sheet, tables with the classic joints and non-programmable pocket calculators are authorised.**

The various parts are independent. The answer sheet **must** be handed in (*even if not completed*).

# MECHANICS TEST 1 - SWELL SIMULATOR

*Marking scale (indicative): Part A /5 ; Part B /10 ; Part C /5*

**Context:**

The mechanical system under consideration is used to generate swell in order to test the stability of hulls in basins for scale models (Figs. 1&2). Waves are generated by flaps which are agitating a certain quantity of water and produce multi-directional swell. In this problem, however, the analysis will be restricted to a single flap in a canal producing unidirectional swell (Figure 3).

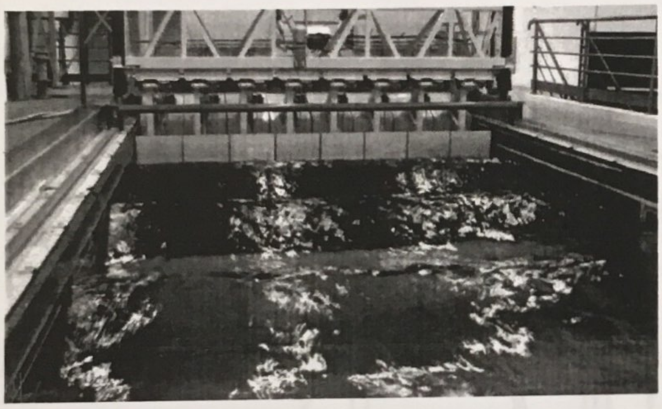

**Figure 1 : multi-directional swell basin.**

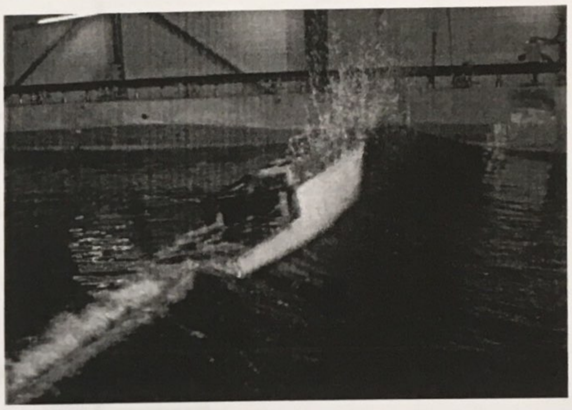

**Figure 2 : Hull submitted to unidirectional swell.**

**Description:** (see Figure 4)

To generate waves, a motor operates the system in Fig. 4 including a flap (solid 5) which agitates the water in the basin. The motor (1) rotation is transmitted to an oscillating arm (3) via a connecting rod (2). The periodic motion of the flap (5) guided by rod (4) is produced by the oscillations of arm (3).

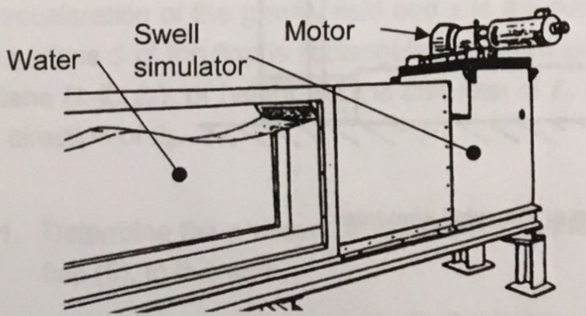


**Figure 3 : Unidirectional swell canal.**

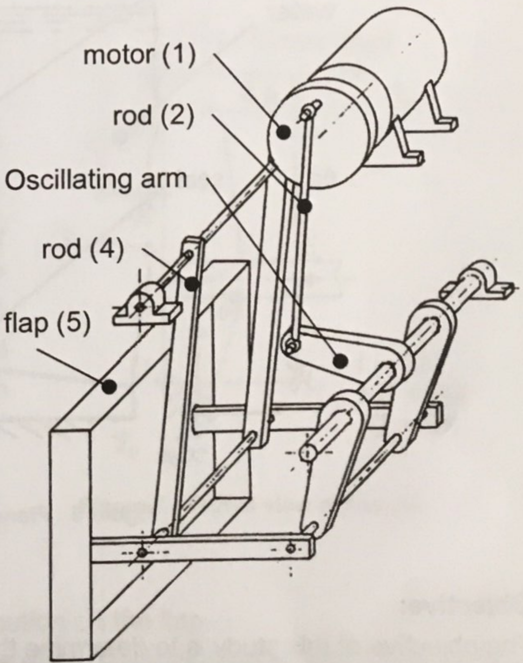


**Figure 4: Swell simulator**

**Hypotheses:** (see Figure 5)

All the joints are perfect (no friction).

The weight of all the parts is neglected except that of the flap (solid 5).

The mechanical actions by the seals are neglected compared with the other mechanical actions to be considered.

The system is, to a large extent, symmetrical with respect to plane $(0, \overrightarrow{x_0}, \overrightarrow{y_0})$ so that the hypothesis of planar problem can be used **in part B**.

**Modelling:** (see Figure 5 and 6)

The joints at $O$, $C$, $D$, $E$, and $F$ are modelled by revolute joints of axis $\vec{x}_0$.

The joints at $A$ and $B$ are modelled as spherical joints of centres $A$ and $B$ respectively.

Gravity is such that $\vec{g} = -g\vec{z}_0$.

The weight of the flap (solid 5) is $\|\vec{P}\| = 600\ N$ and its centre of gravity is G.

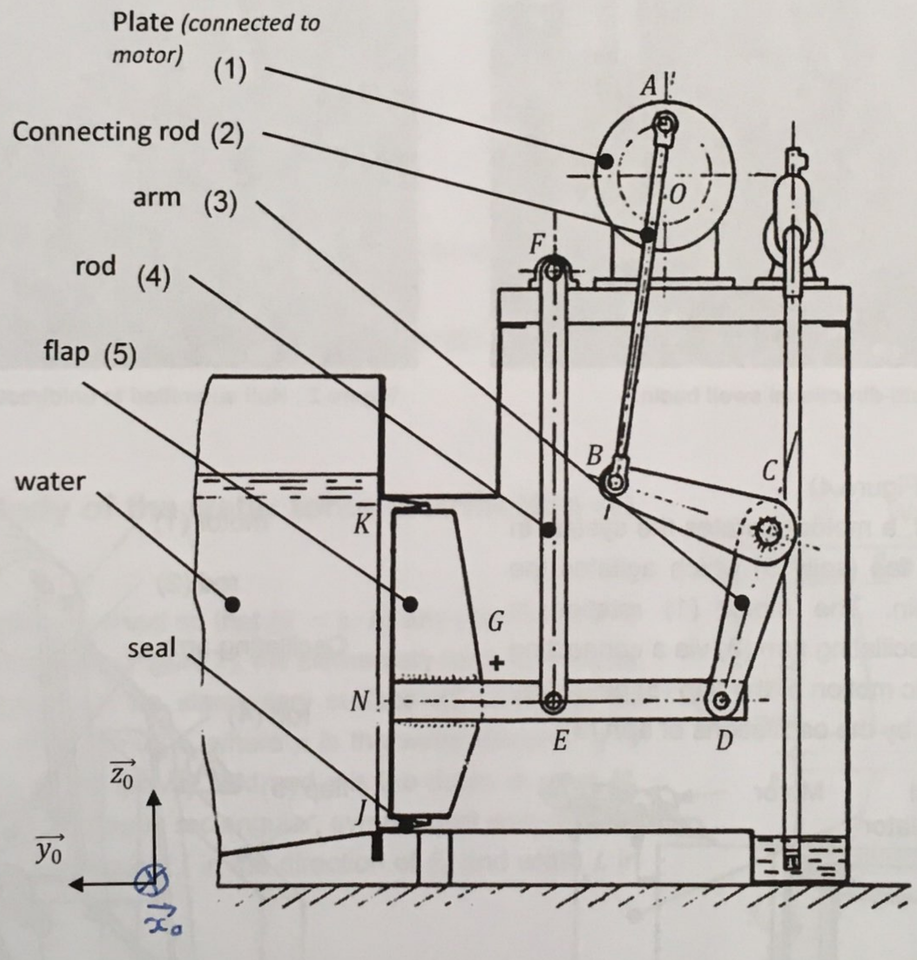


Figure 5 : Planar representation of the swell simulator

**Objective:**

The objective of this study is to determine the couple $\overrightarrow{C_m} = C_m\,\overrightarrow{x_0}$ delivered by the motor in order to maintain static equilibrium in the position defined in Figure 6, i.e., before water is moved in the basin.

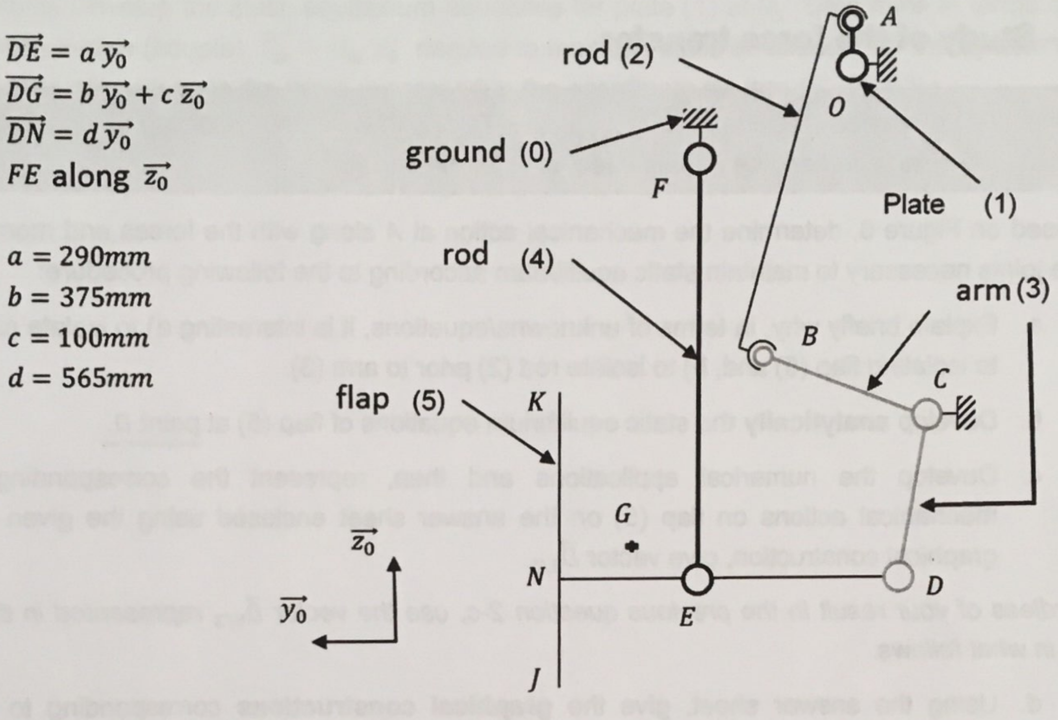


Figure 6 : Schéma cinématique du batteur

*NB : arm (3) comprises the two branches BC and BD which are rigidly connected (same solid).*

## Part A – Study of the water force distribution on the flap

Flap (5) is totally immersed so that $IK = h$. At any point $M$ ( $\overrightarrow{IM} = x\vec{x}_0 + z\vec{z}_v$ ) of surface $S$ (Figure 7), the elementary force exerted by the water pressure on the elementary surface $dS$ controlled by point $M$ reads $d\vec{F} = \rho g z\, dS\, \vec{y}_v$ where $\rho$ is the water density, $g$ is the acceleration of the gravity field and $z$ is the depth at point $M$. The surface $S$ of the flap is rectangular, symmetrical with respect to plane $(I, \vec{y}_v, \vec{z}_v)$, of height $l$ in the direction of $\vec{z}_v$ and width $L$ in the direction of $\vec{x}_0$.

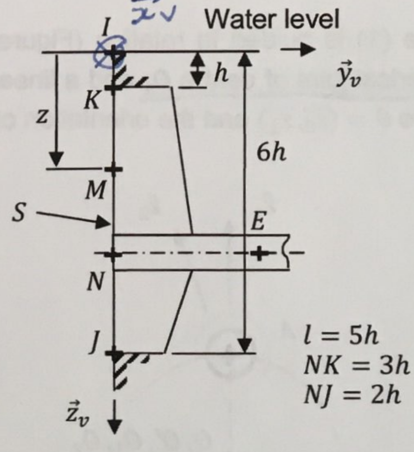


Figure 7 : Partial view of flap (5)

1. Determine the pressure force wrench exerted by water on flap (5), to this end :
   a. Represent schematically the water pressure distribution on the flap.
   b. Determine analytically the sum and moment of the water force wrench on flap (5) at point I, express the results in the coordinate system $(\vec{x}_0, \vec{y}_v, \vec{z}_v)$.

c. Determine the coordinates of the thrust centre $Q$ ( $\vec{IQ} = x_Q\vec{x}_0 + z_Q\vec{z}_v$ ), point on surface $S$ such that the resulting moment of the water force distribution on flap (5) is nil.

## Part B – Study of the force transfer

Regardless of your previous results, take $\vec{Q}_{\text{water}/5} = -Y_Q\,\vec{y}_0$ with $Y_Q = 1400\,N$ acting at the thrust centre $Q$ such that $Q \equiv N$ to represent the overall effect of water on flap (5)

**2.** Based on Figure 6, determine the mechanical action at $A$ along with the forces and moments in all the joints necessary to maintain static equilibrium according to the following procedure:

a. Explain briefly why, in terms of unknowns/equations, it is interesting a) to isolate rod (4) prior to isolating flap (5) and, b) to isolate rod (2) prior to arm (3).

b. Develop **analytically** the static equilibrium equations of flap (5) at point $D$.

c. Develop the numerical applications and then, represent the corresponding external mechanical actions on flap (5) on the answer sheet enclosed using the given scale. By graphical construction, give vector $\vec{D}_{3/5}$.

*Regardless of your result in the previous question 2-c, use the vector $\vec{D}_{5/3}$ represented in the answer sheet in what follows.*

d. Using the answer sheet, give the **graphical constructions** corresponding to the static equilibrium of rod (2) and oscillating arm (3).

## Part C – Study of the mechanical actions on the plate connected to the motor

Regardless of the previous results, take $\vec{A}_{2/1} = -Z_A\,\vec{z}_2$ .

Plate (1) is guided in rotation (Figure 8) by two angular-contact ball bearings which are modelled as a spherical joint of centre $O_1$ and a linear-annular joint of axis $(O_2, \vec{x}_0)$. The position of plate (1) is defined by angle $\theta = (\vec{z_0}, \vec{z_1})$ and the orientation of rod (2) is defined by angle $\alpha = (\vec{z_1}, \vec{z_2})$.

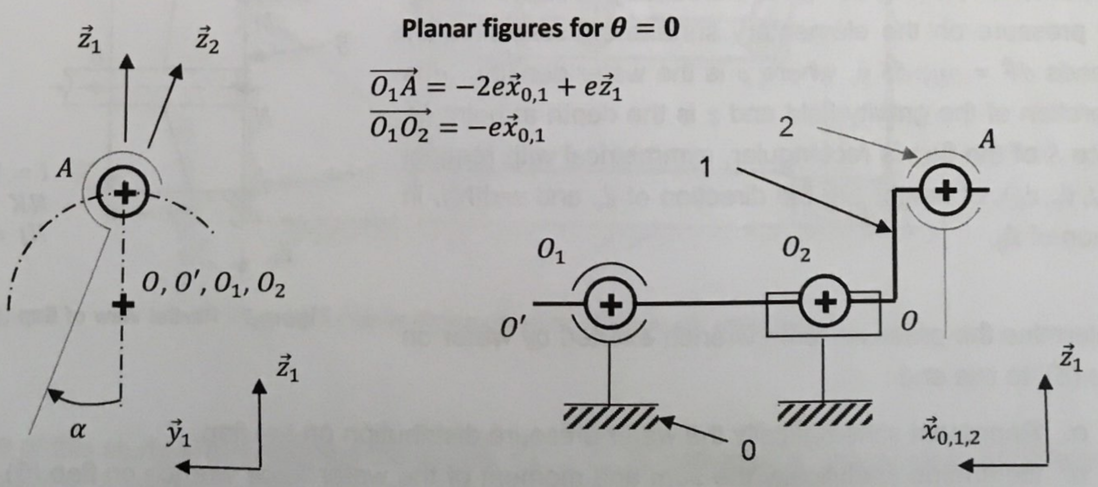


**Figure 8 : Planar representations of plate (1)**

**3.** *Reminder: In this question, the weight of plate (1) is neglected compared with the other mechanical actions.* Develop the static equilibrium equations for plate (1) at $O_1$. Determine in terms of $Z_A$ a) the driving torque (couple) $\overrightarrow{C_m} = C_m\, \overrightarrow{x_0}$ needed to maintain static equilibrium for the system and, b) the mechanical actions in the joints expressed in the coordinate system $(\vec{x}_0, \vec{y}_1, \vec{z}_1)$.