INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

1st cycle - 2nd year - Scan

MECHANICS - TEST N° 4

MONDAY 23RD APRIL 2018 – 1H30 (10H15-11H45)

Authorized documents : Formula sheet (3 pages + table of ususal joints + table of inertia matrices)
non programmable calculator

Marking scheme – 1st part : 10pts ; 2nd part : 10pts.
The two parts are independent.

Study of an anemometer (wind gauge)

The study is about an apparatus used for measuring the wind speed and direction (Fig. ??). It is composed of : a weathercock for giving the wind direction - an anemometer composed of three hemispheric cups.

FIGURE 1 – Picture of a weathercock and an anemometer

1. Mass geometry

The three hollow hemispheres are : identical - equally spaced of 120 ° at the end of three massless bars (Fig. ??) - of negligible thickness, radius R and mass M_H each. The hemisphere H is represented in figure ??, it is associated with the frame R_H such that $(O_H, \vec{x}_H, \vec{z}_H)$ contains the base circle with O_H in its centre.

Questions :

1.1. By using the elemental surface defined in figure ~~??~~ 5 (ring of radius r, coordinate y and width $d\ell$) and with θ integration variable, show that the inertia centre of the hemisphere is located on the axis $(O_H, \vec{y}_H)$ at $\frac{R}{2}$. (**Recall** : area of the sphere surface $S = 4\pi R^2$).

1.2. Give with little calculation the inertia matrix of a hollow sphere of mass M_S , radius R, and negligible thickness, at its centre. (**Hint** : the calculation of the inertia matrix trace (the sum of the three diagonal terms) can help !).

1.3. Give the form *a priori* of the inertia matrix of the hollow hemisphere represented in figure ?? at point O_H in the frame R_H as a function of R and M_H, show that all the inertia moments are equal and deduce their expressions from previous question .

In the next questions, one will use Binet's notations A_H, B_H, C_H, D_H, E_H, F_H for the inertia matrix with simplifications if applicable.

1.4. Determine the inertia matrix for the hemisphere H, at point O_1 anemometer centre (Fig. ??) in the frame R_H. Deduce the inertia matrices for H' et H" at O_1 in their respective frames $R_{H'}$ and $R_{H''}$.

In the next questions, one will use Binet's notations $A_{H,1}$, $B_{H,1}$, $C_{H,1}$, $D_{H,1}$, $E_{H,1}$, $F_{H,1}$ for these inertia matrices with simplifications if applicable.

1.5. Determine the inertia matrix for the hemisphere H' at O_1 in the frame R_H and deduce by analogy (**consequently with no calculation**) the inertia matrix for the hemisphere H" at O_1 in the same frame. **In the next questions, one will use Binet's notations $A_{H',1}$, $B_{H',1}$, $C_{H',1}$, $D_{H',1}$, $E_{H',1}$, $F_{H',1}$, et $A_{H'',1}$, $B_{H'',1}$, $C_{H'',1}$, $D_{H'',1}$, $E_{H'',1}$, $F_{H'',1}$ for these inertia matrices with simplifications if applicable.**

1.6. Give without any calculation, but with clear justification, the position of the anemometer mass centre.

1.7. Give the *a priori* form of the anemometer inertia matrix at its inertia centre and expressed in the frame R_H , then from the previous question deduce the expressions of the matrix components.

2. Kinetics and Dynamics

The kinematic scheme of the measurement system {weathercock + anemometer} is represented in figure ??. It comprises :

- a rigid fixed chassis S_0 , frame $R_0=(O,\vec{x}_0,\vec{y}_0,\vec{z}_0)$
- an anemometer S_1 linked to S_0 by a revolute joint of axis $(O_1,\vec{z}_{0,1})$, *parameter of motion :* $\psi = (\vec{x}_0, \vec{x}_1)$.
- a weathercock S_2 linked to S_0 by a revolute joint of axis $(O_2,\vec{z}_{0,2})$, *parameter of motion :* $\phi = (\vec{x}_0, \vec{x}_2)$.

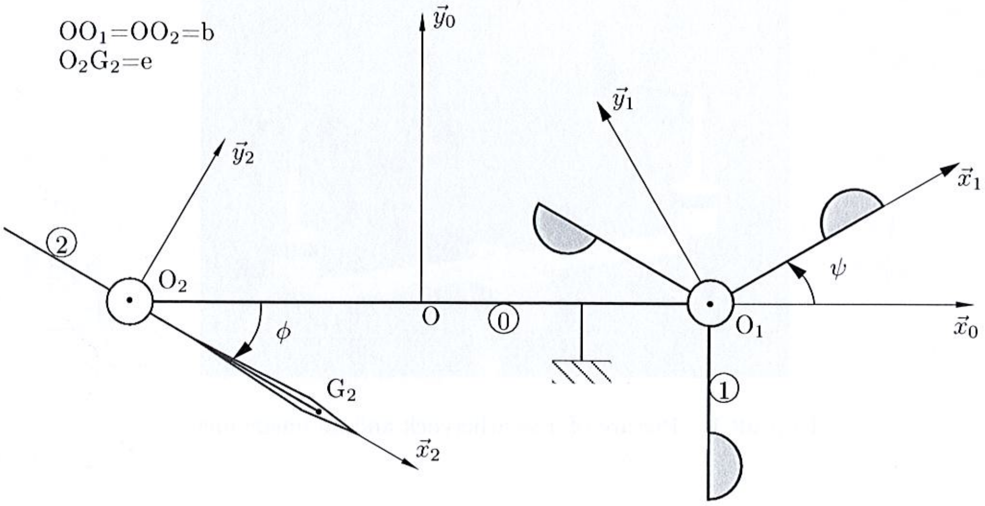

FIGURE 2 – kinematic scheme of the measurement system {anemometer + weathercock}

Mass and Inertia characteristics :

S_1 : mass M_1, inertia centre O_1, the inertia moment with respect to $(O_1,\vec{z}_{0,1})$ is C_1 , $(O_1,\vec{z}_{0,1})$ is a principal axis of inertia.

S_2 : mass M_2, inertia centre G_2, inertia matrix $\bar{\bar{I}}_{G_2,S_2} = \begin{bmatrix} A_2 & 0 & -E_2 \\ 0 & B_2 & 0 \\ -E_2 & 0 & C_2 \end{bmatrix}_2$

Questions :

2.1. Determine the momentum wrench (kinetic wrench) of the anemometer S_1 at O_1 in its motion with respect to R_0.

2.2. Determine the momentum wrench (kinetic wrench) of the weathercock S_2 at O_2 in its motion with respect to R_0.

2.3. Determine the momentum wrench (kinetic wrench)of $\{S_1+S_2\}$ at O in its motion with respect to R_0.

2.4. Determine the dynamic wrench of S_2 at O_2 of the weathercock in its motion with respect to R_0.

2.5. Determine the kinetic energy of $\{S_1+S_2\}$ in its motion with respect to R_0.

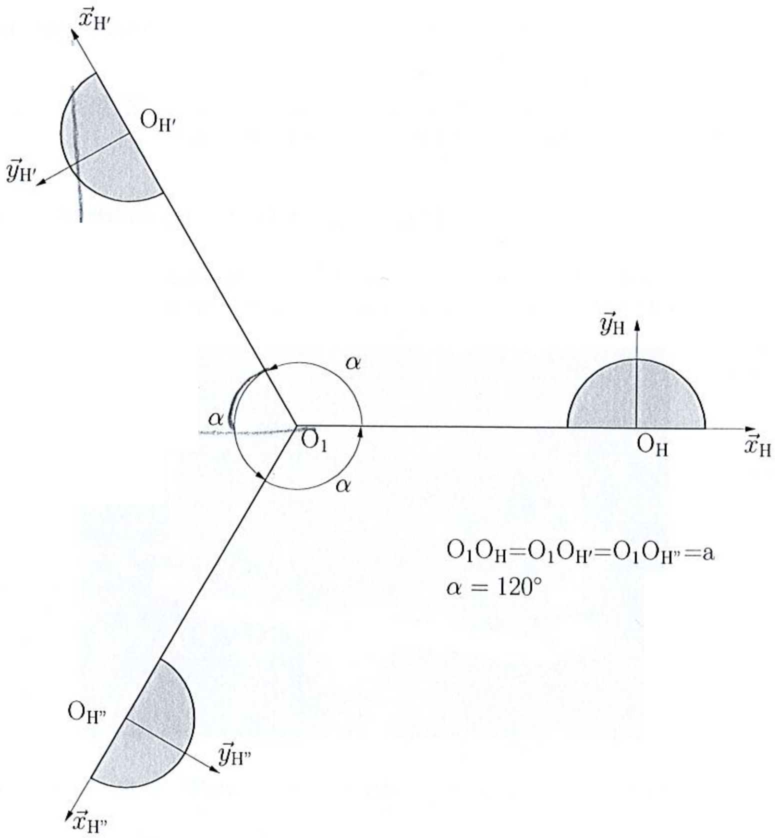

FIGURE 3 – Anemometer scheme

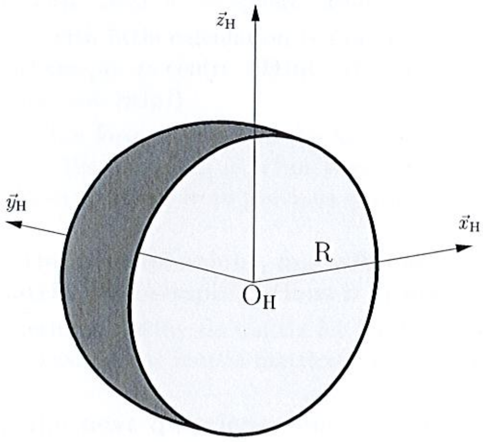

FIGURE 4 – Hollow (empty) hemisphere

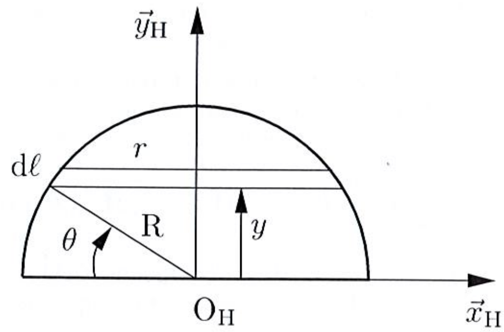

FIGURE 5 – Elemental surface on the hemisphere