INSA INSTITUT NATIONAL
DES SCIENCES
APPLIQUÉES
LYON

FiMi – SCAN 2nd

MECHANICS – Test 1

Monday 19th November 2018 _ 1:30 hour (10:15-11 45)
Authorised material: personal formula sheet (1 page + 1 formula sheet for the joints)
Non-programmable pocket calculator

Indicative mark breakdown: part 1 : /6 ; part 2 : /4 ; part 3 : /6 ; part 4 : /4 4 independent parts

STATIC ANALYSIS OF A BRAKING SYSTEM

The braking system under consideration is shown in Figures 1, 4&5. The kinematic model given in Figure 2 will be used throughout this test.

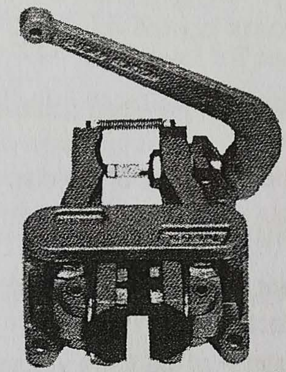

Figure 1 : Braking system (MRH UNICUM)

The brake studied in this paper is aimed at slowing down the rotational speed of disc **18** in Figure 2. It converts the force wrench on lever **17** $\{T(ext \to 17)\} = \begin{Bmatrix} \vec{R}_{ext/17} \\ \vec{0} \end{Bmatrix}_A$ into two mechanical actions on pads **8** and **10** (Figure 2) fixed to disc **18** via

- the pushing rod **14**,
- the jaws **4** and **5**,

MODEL & HYPOTHESES:

The mechanism represented in Figure 2 comprises :

- 5 revolute joints centred at points B, J, L, K and H respectively, whose axes are all in the direction $\vec{z}$
- 2 point contacts at F and G whose (identical) normal vector is $\vec{x}$,
- 1 cylindrical joint of axis $(C,\vec{x})$.
- 1 cylindrical joint of axis $(P,\vec{x})$

The ground is part **1**, considered as fixed to the Galilean frame in this study.

All the joints listed above are supposed to be perfect.

Moreover, two non-perfect planar joints **8**/**18** and **10**/**18** in the plane $(P,\vec{y},\vec{z})$ are considered; they are centred at M and N respectively (Figure 2).

All the weights are neglected compared with the other mechanical actions.

The analysis is decomposed into four independent parts:

- The calculation of the sum and moment about point P of the wrench equivalent to the total action of the pads **10** and **8** on the braking disc **18**.
- The equilibrium of disc **18**.
- The equilibrium of jaw **4** in the particular case when the resisting torque on the disc is nil : Cr=0.
- The graphical analysis of the system equilibrium in the same condition as above.

Part I – Braking force wrench (6 points)

Disc **18** is braked by the action of the two pads **8** and **10**. Each pad is in contact with the disc on a circular sector of angular amplitude 2β, of inner radius R_i and outer radius R_e (Figure 3).
The elemental mechanical actions exerted on disc **18** by pads **8** and **10** at any points M' and N' on the surfaces of contact are respectively :

$$d\vec{F}_{8/18} = d\vec{N}_{8/18} + d\vec{T}_{8/18} = p\,dS\,\vec{x} + f\,p\,dS\,\vec{v}$$
$$d\vec{F}_{10/18} = d\vec{N}_{10/18} + d\vec{T}_{10/18} = -p\,dS\,\vec{x} + f\,p\,dS\,\vec{v}$$

where p (contact pressure) and f (friction coefficient) are two positive constants, $\vec{v}$ is defined in Figure 3.

Remark : For the following questions I.1 and I.2, the thickness of the disc is neglected i.e., points M' and N' are supposed to be in the plane $(P,\vec{y},\vec{z})$.

1.1 – Justify, **with no calculation**, why the moment about P of the mechanical actions from the pads **8** and **10** on disc **18** has no component in the $\vec{y}$ *and* $\vec{z}$ directions: $\vec{M}(P).\vec{y} = \vec{M}(P).\vec{z} = 0$.

1.2 – Determine the sum and the moment about point P of the force wrench exerted by the two pads on the disc.

Indication : Use the polar coordinates. Note that vector $\vec{v}$ of the local coordinate system varies with the position of points M' and N' such that $\vec{v} = -\sin\theta\vec{y} + \cos\theta\vec{z}$.

Part II – Three-dimensional analytical static analysis (5 points)

Braking is activated and keeps the disc at rest; it is assumed that disc **18** is submitted to a resisting torque $C_r\,\vec{x}$. The objective in this section is to determine the limiting value of torque Cr, defining the limit of sliding between disc **18** and pads **8** and **10.**

Regardless of the results obtains in Part I, use the in equilibrium equations the following mechanical actions $\{T_{8/18}\}$ and $\{T_{10/18}\}$ defined as :

$$\{T_{8/18}\} = \begin{Bmatrix} \vec{R}_{8/18} = p\,S\,\vec{x} + f\,p\,S\,\vec{z} \\ \vec{M}_{8/18}(P) = C\,f\,p\,\vec{x} \end{Bmatrix} \quad \text{and} \quad \{T_{10/18}\} = \begin{Bmatrix} \vec{R}_{10/18} = -p\,S\,\vec{x} + f\,p\,S\,\vec{z} \\ \vec{M}_{10/18}(P) = C\,f\,p\,\vec{x} \end{Bmatrix}$$

where p (contact pressure) and f (friction coefficient) are two positive constants, S is the surface of contact between the pads and the disc; C is a constant homogeneous to a volume.

2.1- Static analysis of disc **18** *(see Figure 2)*
Give the liist of all the external mechanical actions on the disc, use the wrench formalism for each of them and express the corresponding moment about point P.

2.2- Develop the equilibrium equations for disc **18**.

2.3- Determine :
- the mechanical actions in the cylindrical joint of axis $(P,\vec{x})$ between **1** and **18** .
- the expression of the resisting torque C_r in terms of f, p and C.

Part III – Bidimensional analytical static analysis (5 points)

The objective in this section is to connect the braking forces at the pads/disc interfaces to the action of the pushing rod **14** on jaw **4**.

Hypotheses

- No resisting torque on disc **18** : Cr = 0.

  In these conditions, the actions from the pads on the disc have no tangential components $d\vec{T}_{8/18}$ and $d\vec{T}_{10/18}$.

- The mechanical actions associated with spring **2** (to open the brake jaws) are also neglected.

The problem is planar (within $(P,\vec{x},\vec{y})$).

3.1- In view of the modelling hypotheses, give the direction of the resultants $\vec{R}_{14/4}$ *and* $\vec{R}_{14/17}$ Justify.

3.2- Isolating solid **8**, determine the direction of the mechanical action $\vec{R}_{4/8}$.

3.3- Determine the joint forces at points H, K et F in terms of the contact action **18/8** *e.g.* $\vec{R}_{18/8}=-pS\vec{x}$ and in terms of the geometrical data of part **4** given in Figure 2.

Part IV– Graphical statics (4 points)

The objective in this section is to determine by graphical construction the force exerted by the lever 17 on the pushing rod. 14

Hypothesis

As in part III, the mechanical actions associated with spring **2** (to open the brake jaws) are neglected.

4.1 – Using the annex, determine by graphical construction the force exerted by the lever **17** on the pushing rod **14**, knowing the external force $\vec{R}_{Ext./17}$ (represented in the annex). Justify the graphical construction(s) and specify clearly the isolated bodies/sub-systems.

4.2 – Numerical application

scale : 1 cm <-> 200 N.

Give in the annex the amplitude of $\vec{R}_{17/14}$.

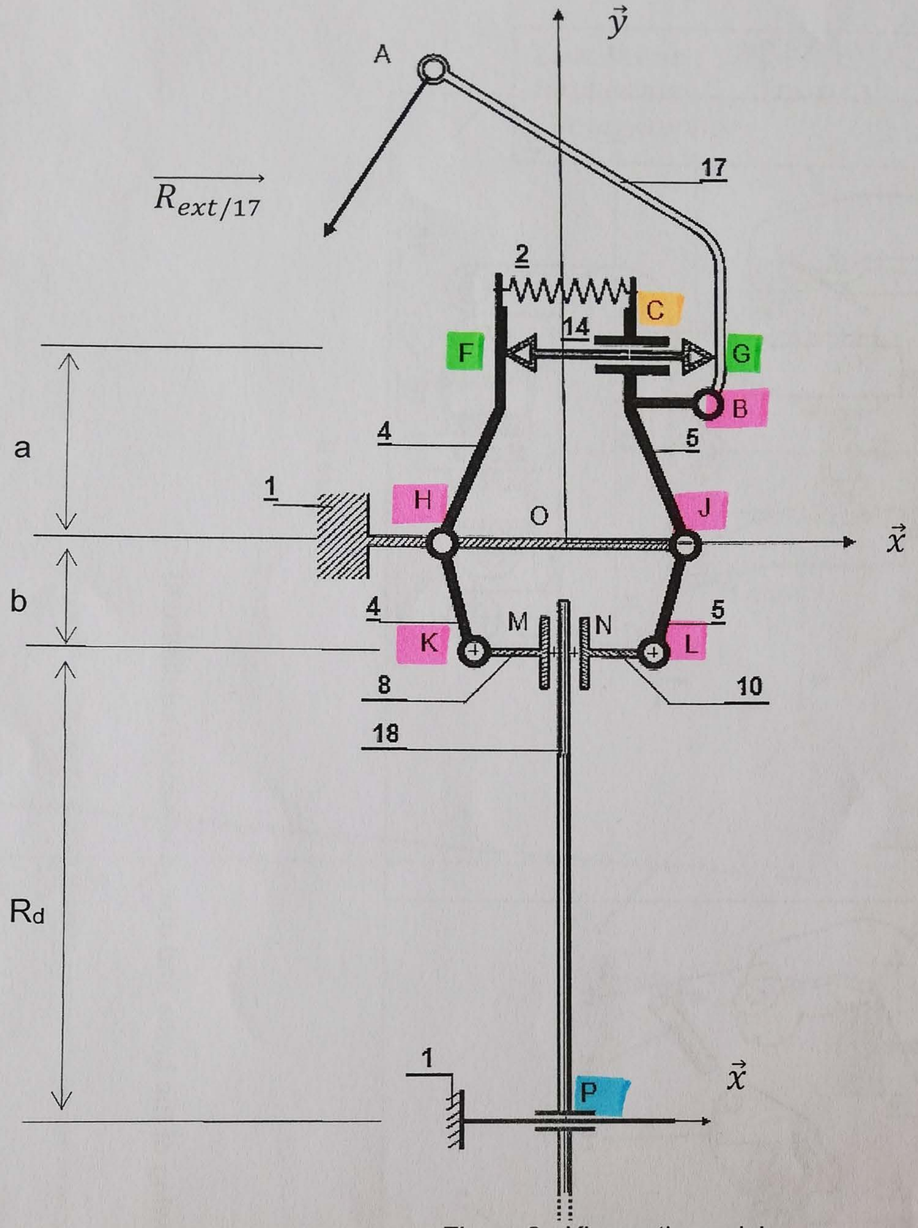

Figure 2 : Kinematic model

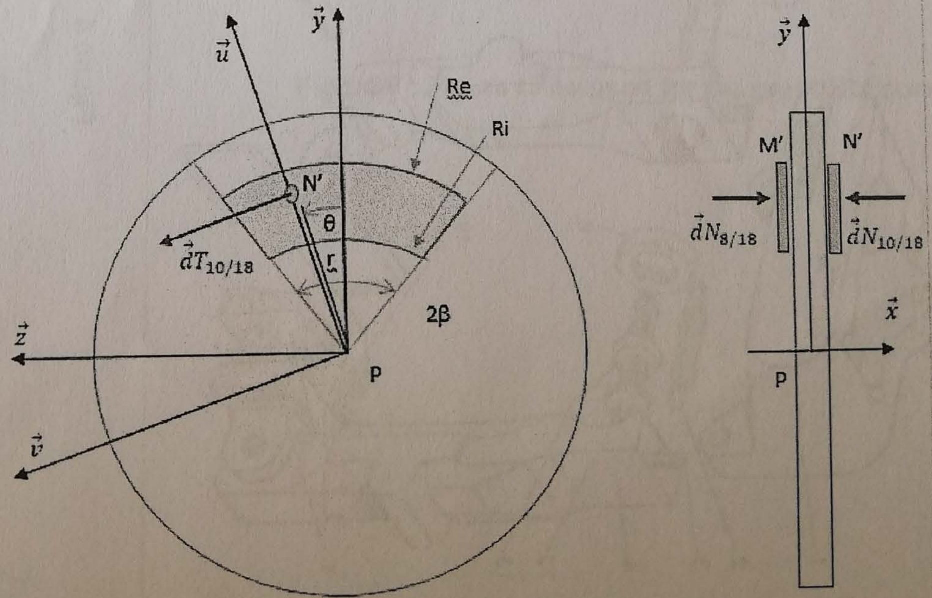

Figure 3 : Schematic representation of the pad/discs interfaces

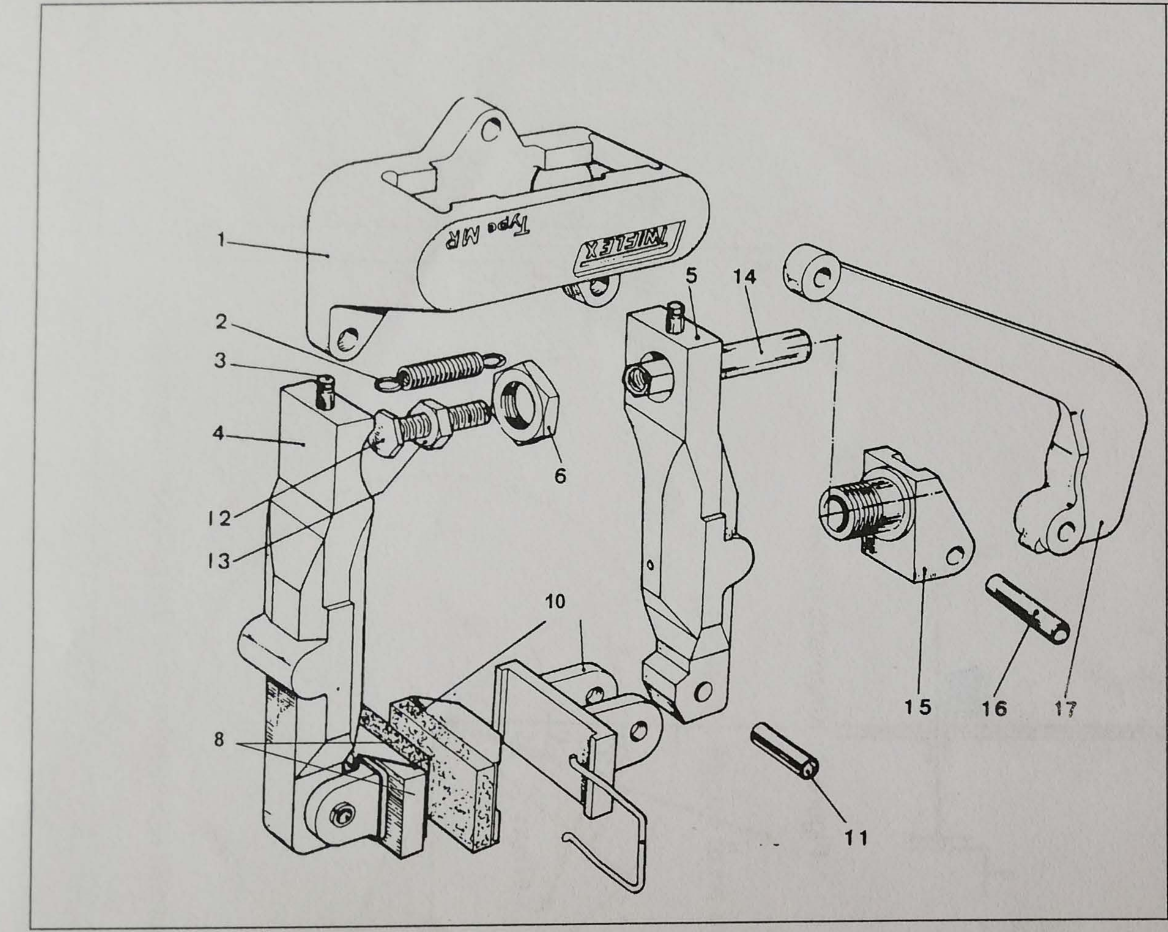

Figure 4

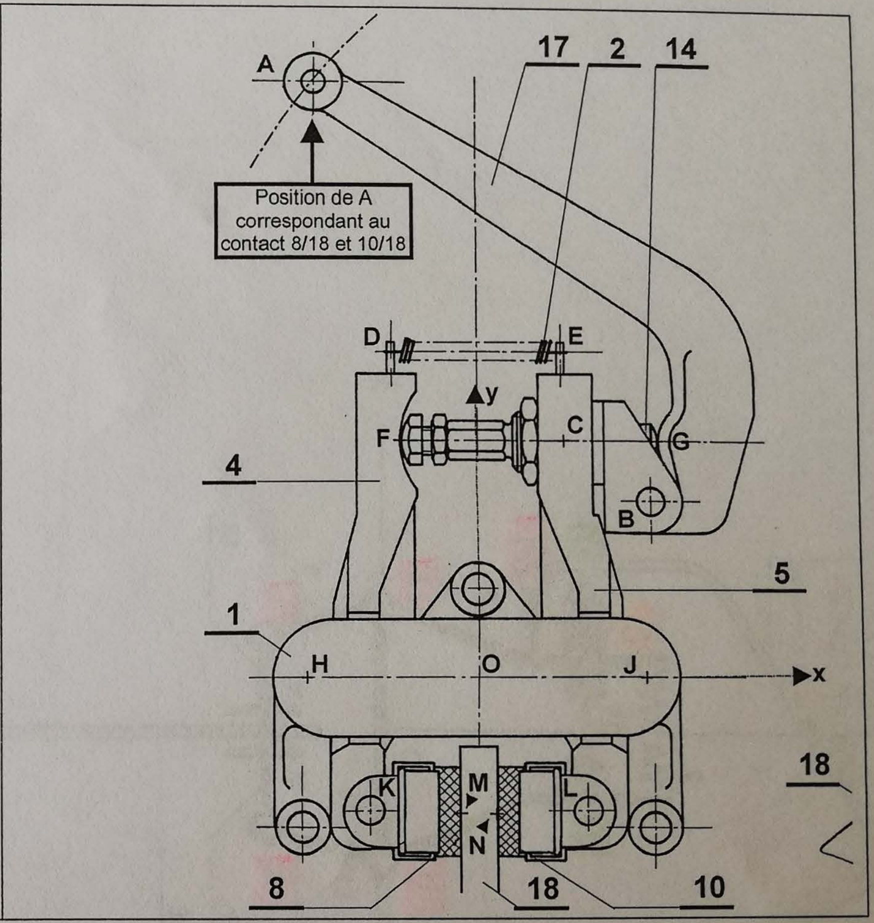

Figure 5

Partial drawing of the brake and disc (position of equilibrium)

NB : The figures are taken from « Mécanique », J.Pailley, C.Reboul et J.Thierry, Editions Educalivre

TO BE HANDED IN

Last Name : SÉRÈMEJ
First name : Loan
Group number : 71

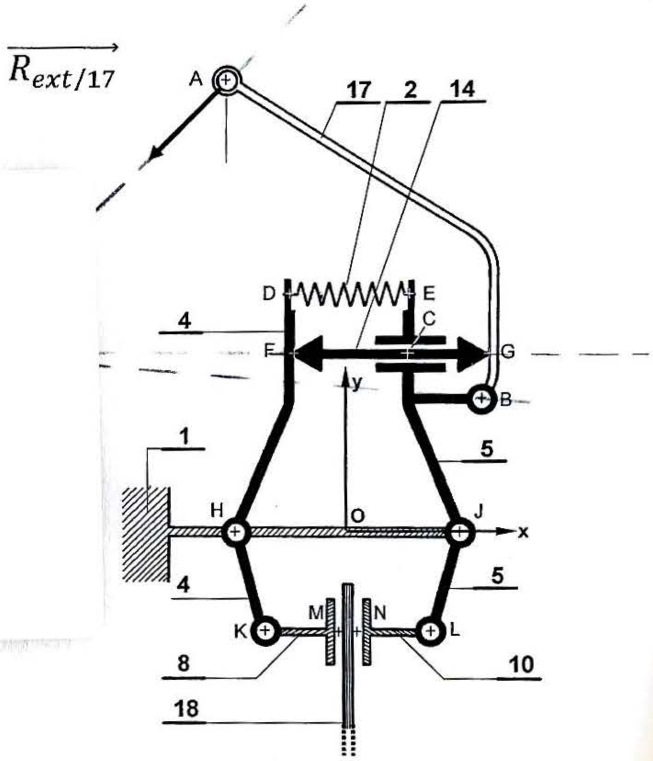

Figure 6 : Figure to be used for the graphical construction

$\left\|\vec{R}_{17/14}\right\| =$ ....