Mechanics – Test# 4
08/04/2019 – 1h30 (10h15 – 11h45)

Authorised documents:
- Personal Formula sheet (3 pages + 1 joint wrench + 1 inertia matrices)
- Non programmable Calculator

Analysis of a Swiss anchor escapement mechanism

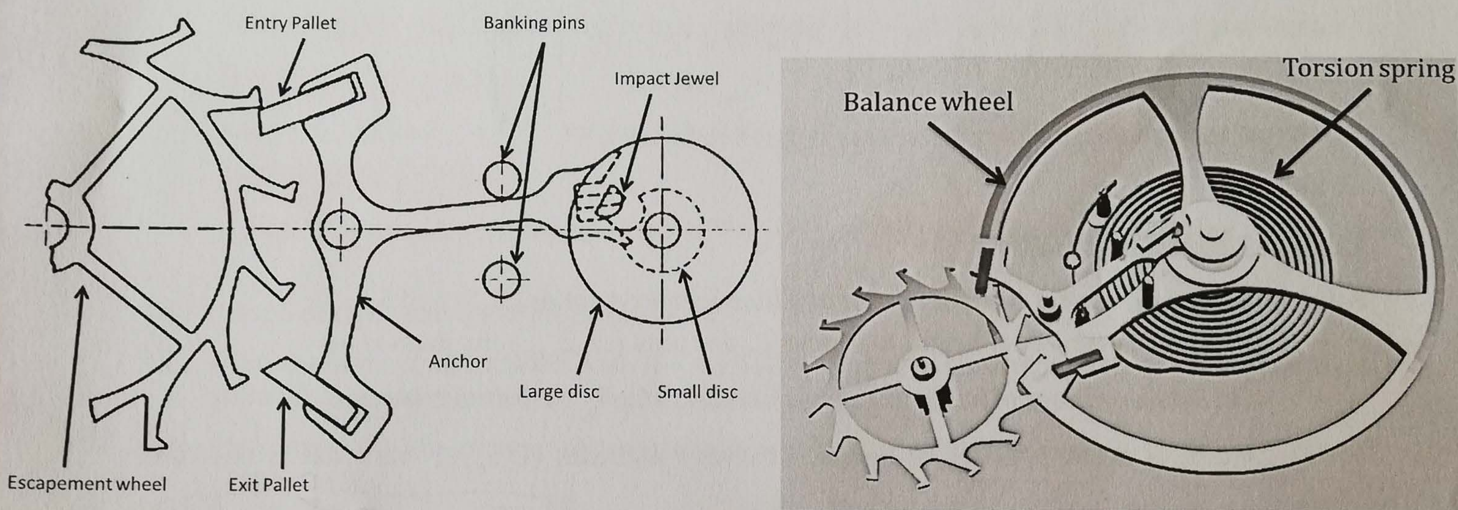


Figure 1 – Swiss anchor escapement mechanism of a watch

System description

The Swiss anchor escapement mechanism (**Figure 1**) is a key component in many mechanical watches as it controls watch accuracy to a large extent.
This mechanism shown in **Figure 2** is operated by the energy stored in a torsion spring R_T of stiffness k (not represented in figure2) which makes solid S_1 oscillate periodically by an angle $\pm\alpha$ around axis $(O_1,\vec{z}_0)$ at a frequency of a couple of Hz. Solid S_1 comprises a large disc S_A, a small disc S_B and an impact jewel S_C (**Figure 3**). It is linked to the watch frame S_0 by a revolute joint of axis $(O_1,\vec{z}_{0,1,2})$ and parameter $\alpha=(\vec{x}_0,\vec{x}_1)=(\vec{y}_0,\vec{y}_1)$;
The $\pm\alpha$ oscillations of S_1 induce a periodic push of the anchor S_2 by an angle $\pm\beta$ because of the point contact at A between the impact jewel S_C and S_2. The mechanical actions transmitted by the point contact joint between S_1 (via S_c) and S_2 at point A are expressed by the wrench $\left\{F_{2\to1}\right\}_A$.
The anchor S_2 is linked to the watch frame S_0 by a revolute joint of axis $(O_2,\vec{z}_0)$ and parameter $\beta=(\vec{x}_0,\vec{x}_2)=(\vec{y}_0,\vec{y}_2)$.
The periodic oscillation of anchor S_2 permits releasing the rotation of the escapement wheel R_E (not modeled) which is in contact with S_2 at point D. The contact forces at point D are known and expressed by the wrench $\left\{F_{R_E\to2}\right\}_D$.

Hypotheses :

- the frame $R_0 = (O_1, \vec{x}_0, \vec{y}_0, \vec{z}_0)$ is a Galilean frame,
- the problem is considered as planar,
- the gravity is downwards along $\vec{z}_{0,1,2}$,
- all the joints are perfect,
- the torsion spring R_T acts onto S_1 at point O_1. It has a constant stiffness k and its angle at rest is α_0. Its mass is neglected and its force wrench is denoted $\left\{F_{R_T \to 1}\right\}_{O_1}$.
- The contact force wrench between S_1 and S_2 at point A reads:

$$\left\{F_{2\to 1}\right\}_A = \left\{ \begin{array}{c} \vec{R}_{2\to 1} = X_{21}\vec{x}_1 + Y_{21}\vec{y}_1 \\ \vec{M}_{2\to 1}(A) = \vec{0} \end{array} \right\}_A$$

All the geometrical parameters are defined in figure 2.

Part 1 : Mass Geometry

Solid S_1 of mass m_1 can be decomposed into 3 homogeneous solids, S_A, S_B and S_C:

- solid S_A is a cylinder of height h_A, radius R_A and axis $(O_1, \vec{z}_{0,1,2})$. Its mass is m_A.
- solid S_B is a cylinder of height h, radius r and axis $(O_1, \vec{z}_{0,1,2})$. Its mass is m_B.
- solid S_C is a half-cylinder of height h, radius r and axis $(C, \vec{z}_{0,1,2})$. Its mass is m_C. The coordinates of the mass center G_C of solid S_C are given by : $\boxed{\overrightarrow{O_1G_C} = (+L_C - \frac{4\cancel{R}\,r}{3\pi})\vec{y}_1 + \frac{h}{2}\vec{z}_1}$.

Solid S_2 has negligible thickness, its mass center is G_2 $\boxed{\overrightarrow{O_2G_2} = x_{G_2}\vec{x}_2}$, its mass is m_2.

<u>Do not calculate the masses of the solids.</u>

The inertia matrices of S_1, S_2 and S_C are given as:

$$\bar{\bar{I}}_{O_1,S_1} = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & B_1 & -D_1 \\ 0 & -D_1 & C_1 \end{bmatrix}_{R_1} \quad \bar{\bar{I}}_{G_2,S_2} = \begin{bmatrix} A_2 & -F_2 & 0 \\ -F_2 & B_2 & 0 \\ 0 & 0 & C_2 \end{bmatrix}_{R_2} \quad \bar{\bar{I}}_{G_C,S_C} = \begin{bmatrix} A_{SC} & 0 & 0 \\ 0 & B_{SC} & 0 \\ 0 & 0 & C_{SC} \end{bmatrix}_{R_1}$$

1.1 Justify the inertia matrix form of solid S_2 at point G_2 expressed in basis R_2.

1.2 Justify the inertia matrix form of solid S_1 at point O_1 expressed in basis R_1.

1.3 Determine the coordinates of the mass center G_1 of solid S_1 in R_1, express $\overrightarrow{O_1G_1}$.
For the next questions, these coordinates will be denoted: $\overrightarrow{O_1G_1} = x_{G_1}\vec{x}_1 + x_{G_1}\vec{y}_1 + x_{G_1}\vec{z}_1$

Study of solid $S_1 = \{S_A \cup S_B \cup S_C\}$:

1.4 Determine the inertia matrix of the small disc S_B at O_1. Detail the expression of the matrix terms.

For the next questions, one will use Binet's notations for the matrix terms: A_{SA}, B_{SA}, C_{SA}, D_{SA}, E_{SA}, F_{SA}

1.6 Determine the inertia matrix of the impact jewel S_C at O_1. *Use the terms of* $\bar{\bar{I}}_{G_C,S_C}$.

1.7 Determine the inertia matrix of S_1 at O_1. Detail the expression of the matrix terms.

For the next questions, one will use Binet's notations for the matrix terms: A_1, B_1, C_1, D_1, E_1, F_1

Part 2 : Kinetics and dynamics

The kinematic scheme of the Swiss escapement anchor mechanism is represented in Figure 2. It is composed of:

- the watch frame S_0 (watch housing), $R_0 = (O_1, \vec{x}_0, \vec{y}_0, \vec{z}_0)$,
- solid S_1(mass m_1) linked to S_0 by a revolute joint of axis $(O_1, \vec{z}_{0,1,2})$ and parameter $\alpha = (\vec{x}_0, \vec{x}_1) = (\vec{y}_0, \vec{y}_1)$,
- solid S_2 linked to S_0 by a revolute joint of axis $(O_2, \vec{z}_{0,1,2})$ and parameter $\beta = (\vec{x}_0, \vec{x}_2) = (\vec{y}_0, \vec{y}_2)$,
- a torsion spring R_T (not represented in Figure 2), of stiffness k, acting on S_1.

2.1 Graph of links, change of basis diagrams.

2.2 Give the wrench associated with the torsion spring R_T mechanical actions onto solid S_1 at point O_1 expressed in the coordinate system R_1.

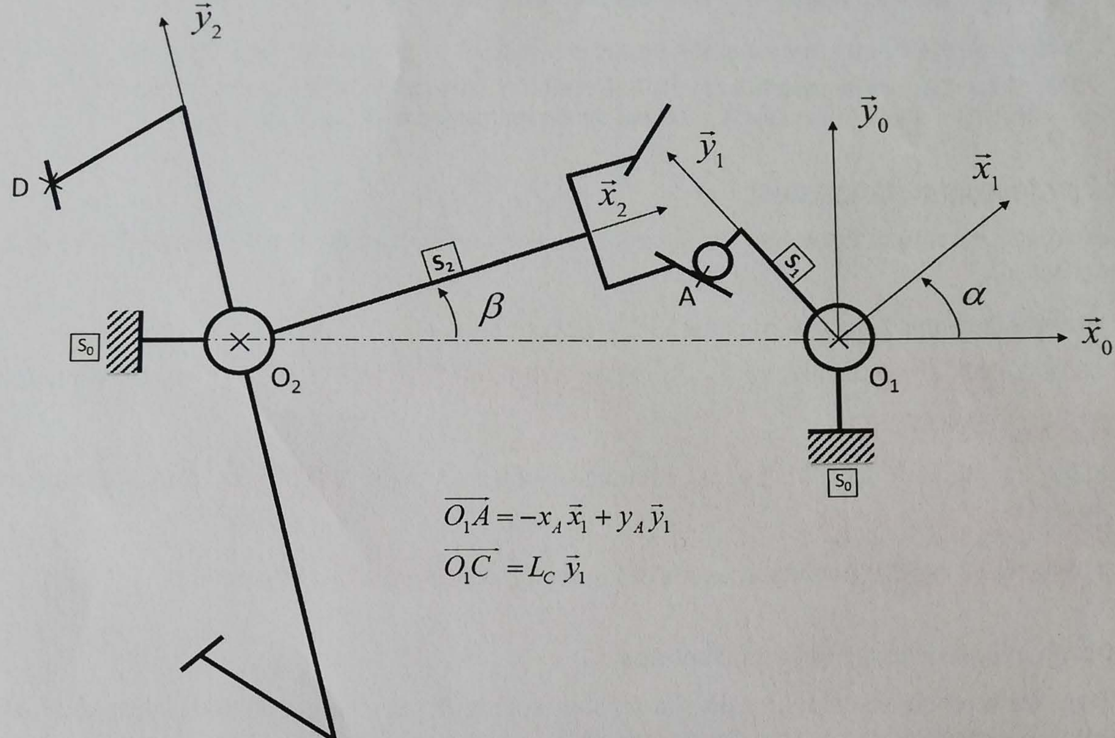
? **2.3** Determine the kinetic wrench (momentum wrench) of solid S_1 at point O_1 in its motion with respect to R_0.

? **2.4** Determine the dynamic wrench of solid S_1 at point O_1 in its motion with respect to R_0.

2.5 Write the dynamic sum theorem for solid S_1.

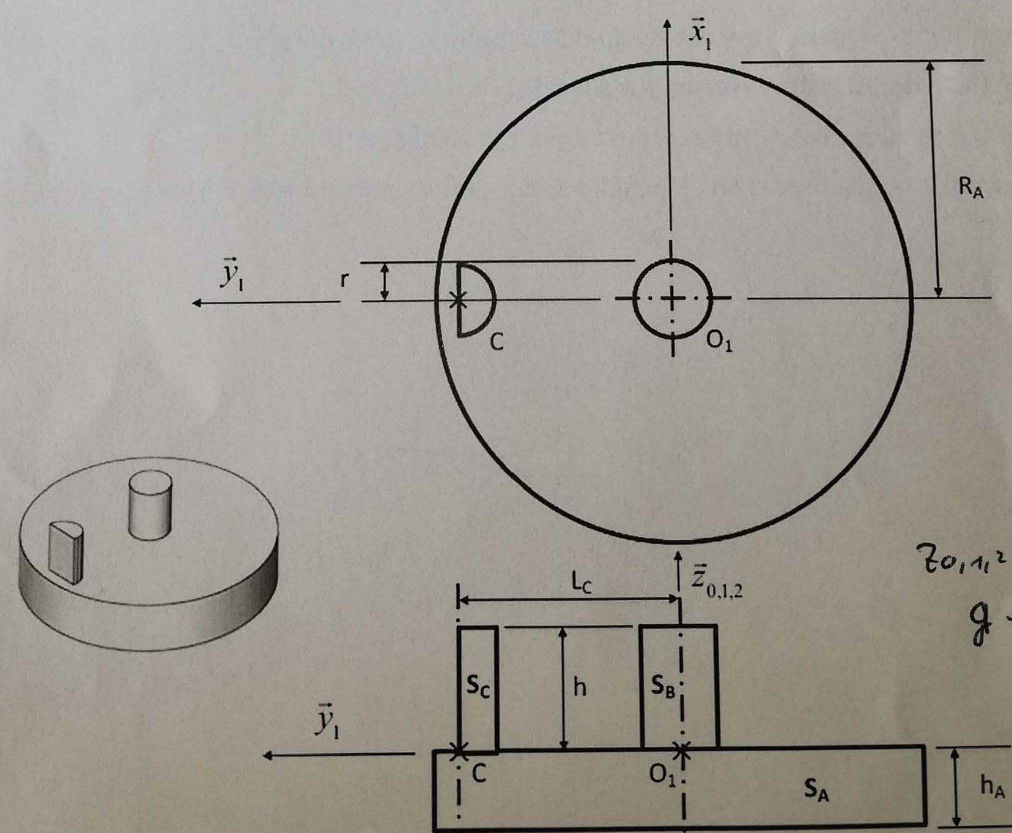
2.6 Write the dynamic moment theorem applied to solid S_1 at O_1

2.7 Determine the dynamic wrench of solid S_2 at point O_2 in its motion with respect to R_0.



$$\overrightarrow{O_1A} = -x_A\,\vec{x}_1 + y_A\,\vec{y}_1$$
$$\overrightarrow{O_1C} = L_C\,\vec{y}_1$$

Figure 2 : Schematic representation of S_1 and S_2



$z_{0,1,2}$ is vertical upward
g is downward

Figure 3 : Solid $S_1 = \{S_A \cup S_B \cup S_C\}$