

A personal 2-page (A4) formula sheet, the tables of classic joints and non-programmable calculators are authorised

Indicative marking scale: A-1 : 5 marks ; A-2 : 5 marks ; B-1 : 5 marks ; B-2 : 5 marks – the sub-parts are independent

## BOWLING PINSETTER

### Description:

A bowling pinsetter is a system used to put back in place the bowling pins after every turn. The proposed study deals with a) the driving system of the table (Figure 1) that collects the pins not knocked down between two throws by the same player and that puts in place new pins for a new player and, b) the plier system on the pinsetter that picks up and moves the pins left standing (Figure 2).

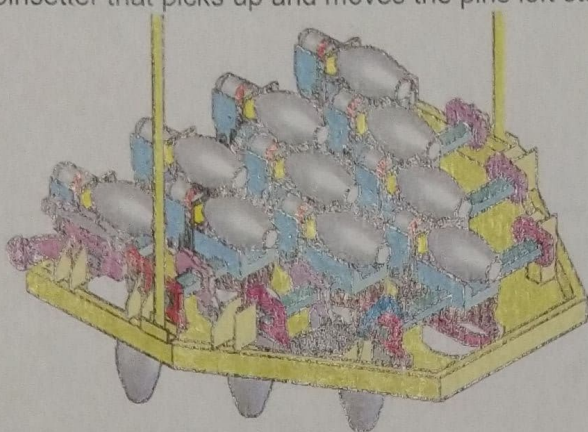


Figure 1 : Representation of a pinsetter

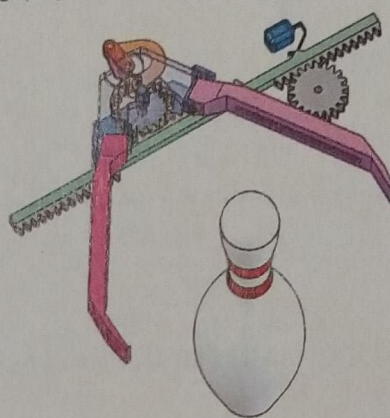


Figure 2 : Plier system to pick up and move pins

### Part A – Pinsetter driving system

**Description :** (see Figure 3)

A motor operates hook (2) via eccentric (1), thus making table (5) move under particular conditions imposed by arm (3) and rod (4).

**Model :** (see answer sheet DR1)

Only the planar  $\vec{x}$  motions in plane  $(O_1, \vec{y}_0, \vec{z}_0)$  of normal  $\vec{x} = \vec{x}_{0,1,2,3,4,5}$  are considered.

- Eccentric (1) is connected to ground (0) by a revolute joint of axis  $(O_1, \vec{x}_{0,1})$  and parameter  $\alpha_1 = (\vec{y}_0, \vec{y}_1)$ ,
- Hook (2) is connected to eccentric (1) by a revolute joint of axis  $(A, \vec{x}_{1,2})$  and parameter  $\beta = (\vec{y}_1, \vec{y}_2)$ ,
- Arm (3) is connected to ground (0) by a revolute joint of axis  $(O_3, \vec{x}_{0,3})$  and parameter  $\alpha_3 = (\vec{y}_0, \vec{y}_3)$ ,
- Rod (4) is connected to ground (0) by a revolute joint of axis  $(O_4, \vec{x}_{0,4})$  and parameter  $\alpha_4 = (\vec{y}_0, \vec{y}_4)$ ,
- Pinsetter (5) is connected to arm (3) by a revolute joint of axis  $(C, \vec{x}_{3,5})$  and parameter  $(/R_0!) \alpha_5 = (\vec{y}_0, \vec{y}_5)$ .

Moreover,

- Hook (2) is also connected to Arm (3) by a revolute joint of axis  $(B, \vec{x}_{2,3})$  with no parameter.
- Rod (4) is also connected to Pinsetter (5) by a revolute joint of axis  $(D, \vec{x}_{4,5})$  with no parameter.



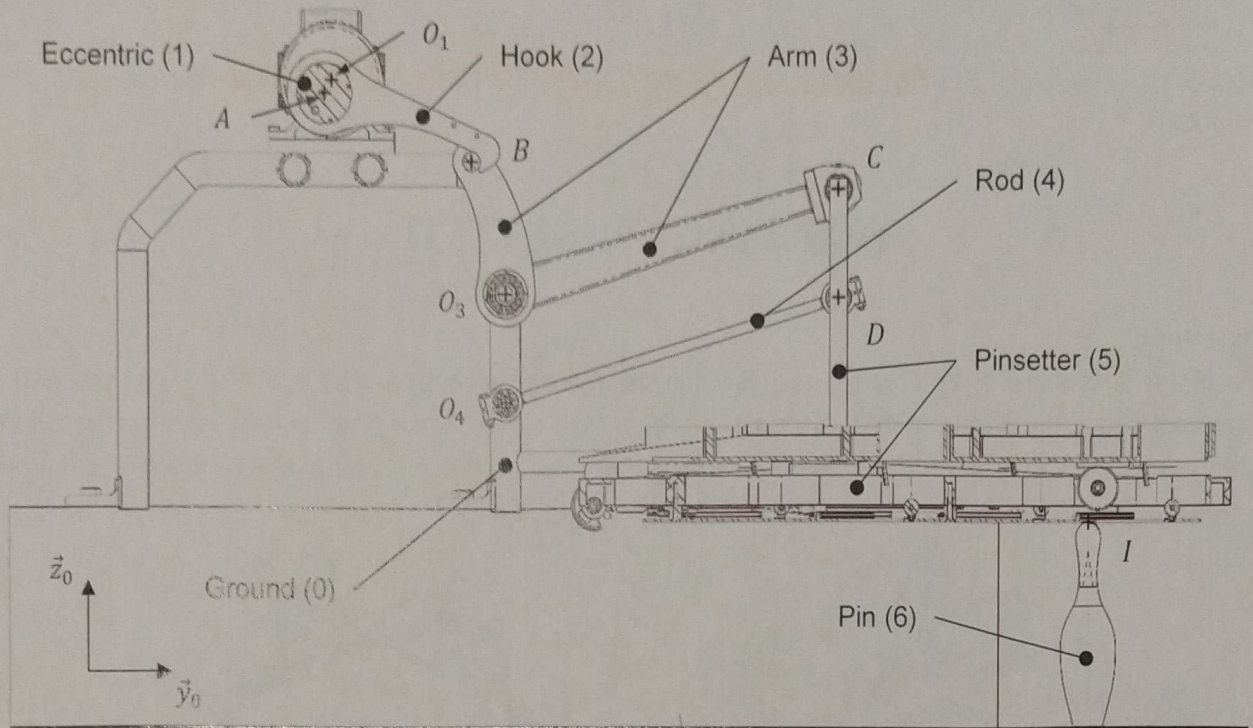


Figure 3 : Pinsetter in intermediate position

### 1) Graphical kinematics of the driving system

$\vec{V}(A/0)$  is given. The graphical constructions should be drawn on answer sheet DR1. **Justify your drawings.**

- A.1. Give the position of the instant centre of motion 2/0 denoted  $I_{20}$  and deduce  $\vec{V}(B/0)$ .
- A.2. Determine graphically  $\vec{V}(C/0)$ .
- A.3. Define the trajectories of points  $C$  and  $D$  with respect to  $R_0$ , characterise motion 5/0 and draw  $\vec{V}(D/0)$  and  $\vec{V}(E/0)$ .
- A.4. The system specifications impose  $\|\vec{V}(E, 5/0)\| = 0,15 \text{ ms}^{-1}$  in order to limit table (5) impact speed at  $E$  against the pins moved from their initial positions. Deduce the maximal rotational speed (in rpm) of eccentric (1) for  $e = 100 \text{ mm}$ .

### 2) Analytical kinematics of the driving system

It is assumed that the constraint equations generated by the revolute joints with no parameter between (2) and (3), (4) and (5) are known which, amongst other equations, lead to  $\alpha_4 = \alpha_3$  and  $\alpha_5 = 0$ .

- A.5. Give the expressions of  $\vec{V}(B/0)$  and the acceleration  $\vec{A}(B/0)$  in terms of  $\alpha_1$  and  $\beta$  and their time-derivatives.
- A.6. Determine velocity  $\vec{V}(C/0)$  and acceleration  $\vec{A}(C/0)$  in terms of  $\alpha_3$  and its time derivatives.
- A.7. Determine the kinematic wrench (screw) for motion 5/0 and calculate acceleration  $\vec{A}(E, 5/0)$ . Specify the nature of this motion.



The notations, labels and coordinate systems in parts A and B have no link.

## Part B – Study of the plier system

**Description :** (see Figure 4)

Axis (3) is moved by the crank (1) – rod (5) system which operates jaw (2) of the plier via the connecting rod (4). Because of symmetry, only one of the two plier jaws is represented. A pinion (7) - rack (8) pair controls the rotation of wheel (1) which opens or closes the plier.

**Modelling :**

Only the planar motions in plane  $(O_1, \vec{x}_0, \vec{y}_0)$ , of normal  $\vec{z} = \vec{z}_{0,1,2,4,5,7}$  are considered.

- wheel (1) of radius  $R_1$  is connected to table (0) by a revolute joint of axis  $(O_1, \vec{z}_{0,1})$  and parameter  $\psi_1 = (\vec{x}_0, \vec{x}_1)$ .
- jaw (2) is connected to table (0) by a revolute joint of axis  $(O_2, \vec{z}_{0,2})$  and parameter  $\psi_2 = (\vec{x}_0, \vec{x}_2)$ .
- axis (3) is connected to table (0) by a prismatic joint of direction  $\vec{y}_0$  whose kinematic parameter is  $y = \overline{O_1 B} \vec{y}_{0,3}$
- rod (4) is connected to axis (3) by a revolute joint of axis  $(B, \vec{z}_{3,4})$  and parameter  $\psi_4 = (\vec{x}_3, \vec{x}_4)$ .
- pinion (7) of radius  $R_7$  is connected to table (0) by a revolute joint of axis  $(O_7, \vec{z}_{0,7})$  and parameter  $\psi_7 = (\vec{x}_0, \vec{x}_7)$ .
- rack (8) is connected to table (0) by a prismatic joint of direction  $\vec{x}_0$  and parameter  $x = \overline{O_1 O_8} \vec{x}_{0,8}$ .

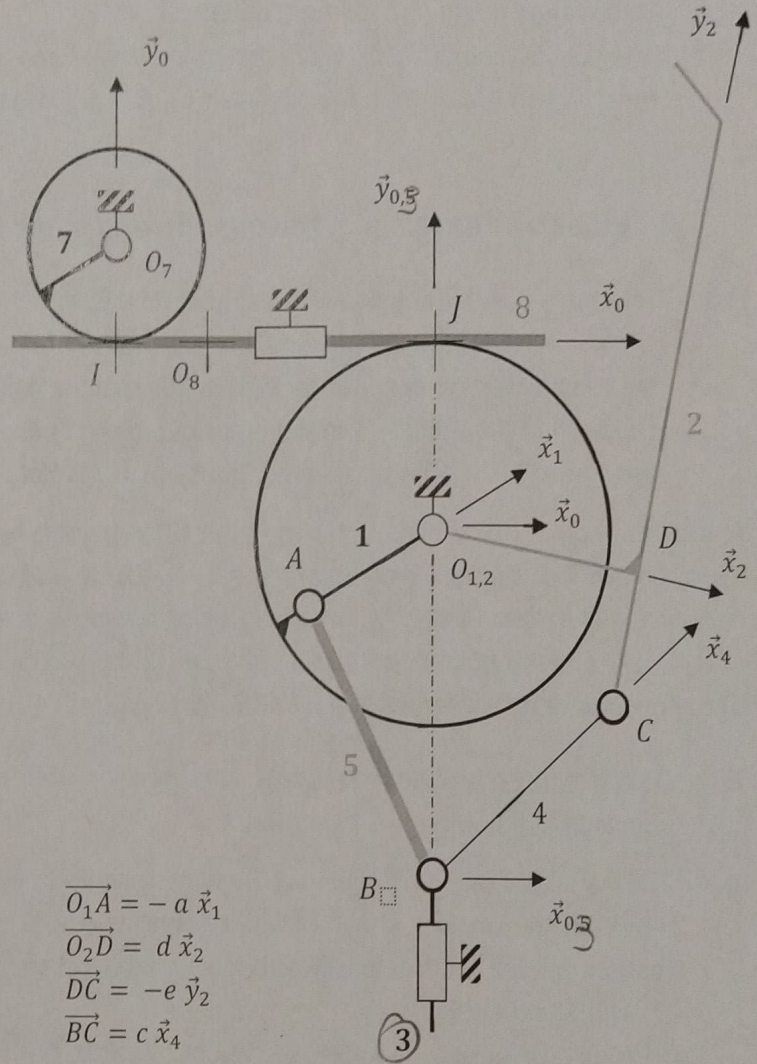


Figure 4 : Kinematic model of the plier

Moreover,

- rod (5) of length  $b$ , connects (3) and (1), no parameter is introduced.
- rod (4) is connected to jaw (2) by a revolute joint of axis  $(C, \vec{z}_{2,4})$  and no parameter.
- pinion (7) is also in contact with rack (8) at point  $I$  with normal  $\vec{y}_0$ , no parameter
- rack (8) is also in contact with wheel (1) at point  $J$  with normal  $\vec{y}_0$ , no parameter



## 1) Geometrical analysis of the plier

- B.1.** Draw the graph of links and the change of basis diagrams.  
**B.2.** Develop the constraint equation(s) associated with the joint between axis (3) and wheel (1) by the intermediate rod (5).  
**B.3.** Develop the constraint equation(s) associated with the joint between rod (4) and jaw (2).

In order to analyse the consequences of the previous constraint equations, one uses the following data in terms of variable  $a$ :  $b = 2a$ ;  $c = a\sqrt{3}$ ;  $d = 3a/2$   $b = 2a$

- B.4.** Determine/ verify the characteristics of the positions « open plier » and « closed plier » :  
 a) The plier is closed for  $\psi_1 = 0$  and  $\psi_2 = 0$ . Determine the expressions of  $y$ ,  $\psi_4$  and  $e$  in this case (mainly in terms of  $a$ ). The negative solution for  $y$  will not be considered and only the solution for  $\psi_4$  in the range 0 and  $\pi/2$  will be retained.  
 b) The plier is open for  $\psi_1 = \pi/2$ . Determine the expression for  $y$  (in terms of  $a$ ), the solution  $y = a$  will be excluded. Verify that  $\psi_4 = \pi/3$  and  $\psi_2 = -\pi/6$  are solutions of the constraint equations.

## 2) Kinematic analysis of the plier driving system

- B.5.** Determine the characteristics of the plier driving system by:  
 a) Using the no-slipping conditions at point  $I$  and  $J$ , determine the rotational speed  $\dot{\psi}_7$  of pinion(7) and the translational speed  $\dot{x}$  of rack (8) in terms of the wheel(1) angular speed  $\dot{\psi}_1$ .  
 b) Calculating the angular variation  $\Delta\psi_7$  of pinion (7) and the position variation  $\Delta x$  of rack (8) between the closed ( $\psi_1 = 0$ ) and open positions ( $\psi_1 = \pi/2$ ) of the plier for  $R_1 = 100\text{mm}$  and  $R_7 = 50\text{mm}$ .

A non-centred pin (6) gets in contact with jaw (2) at point  $K$  before the plier is totally closed as shown in the figure below. Pin (6) is connected to the ground (0) by a planar joint of normal vector  $\vec{z}_0$ , the centre  $O_6$  of the pin cross section is positioned by  $\vec{O}_2O_6 = \lambda\vec{y}_2$  and the point of contact  $K$  with jaw (2) by  $\vec{O}_2K = \lambda\vec{y}_2 + d\vec{x}_2$ . The motion parameters for 6/0 are  $\lambda$  and  $\psi_6 = (\vec{x}_0, \vec{x}_6)$ .

- B.6.** Determine the velocity  $\vec{V}(O_6/0)$  and develop the no-slipping condition at point  $K$ .  
**B.7.** The trajectory of  $O_6$  with respect  $R_0$  is represented by the dotted curve on the answer sheet DR2 such that vector  $\vec{t}$  is the local tangent vector for the position under consideration. Using the answer sheet DR2 :  
 a)  $\vec{V}(C/0)$  is given, deduce the velocities  $\vec{V}(K, 2/0)$  and then  $\vec{V}(K, 6/0)$ . Justify.  
 b) Find by graphical construction  $I_{60}$  the instant centre of rotation for motion 6/0 and draw  $\vec{V}(O_6/0)$ . Explanations are required.  
 c) Determine the pitching and rolling vectors at  $K$ .

