INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

FIMI SCAN 2nd

MECHANICS – Test 1

Monday 18th November 2019 - 1h30 (10h15-11h45)

Authorised: Formula sheets (1 page + 1 sheet with the joints)
Non-programmable calculator

Marking (approximate): part A : 5 marks ; part B : 11 marks ; part C : 4 marks

The 3 parts are independent

STATIC ANALYSIS OF AN AVIO FLOODGATE

Figure 1 shows an example of implantation of a large capacity floodgate in its environment. The floodgate adjusts the fluid flow between reservoirs 1 and 2 through the circular profile deck A of width L, which pivots around axis $(O,\vec{y}_{0,1})$ to close or open the parallelipipedal orifice N and therefore regulates the flow.

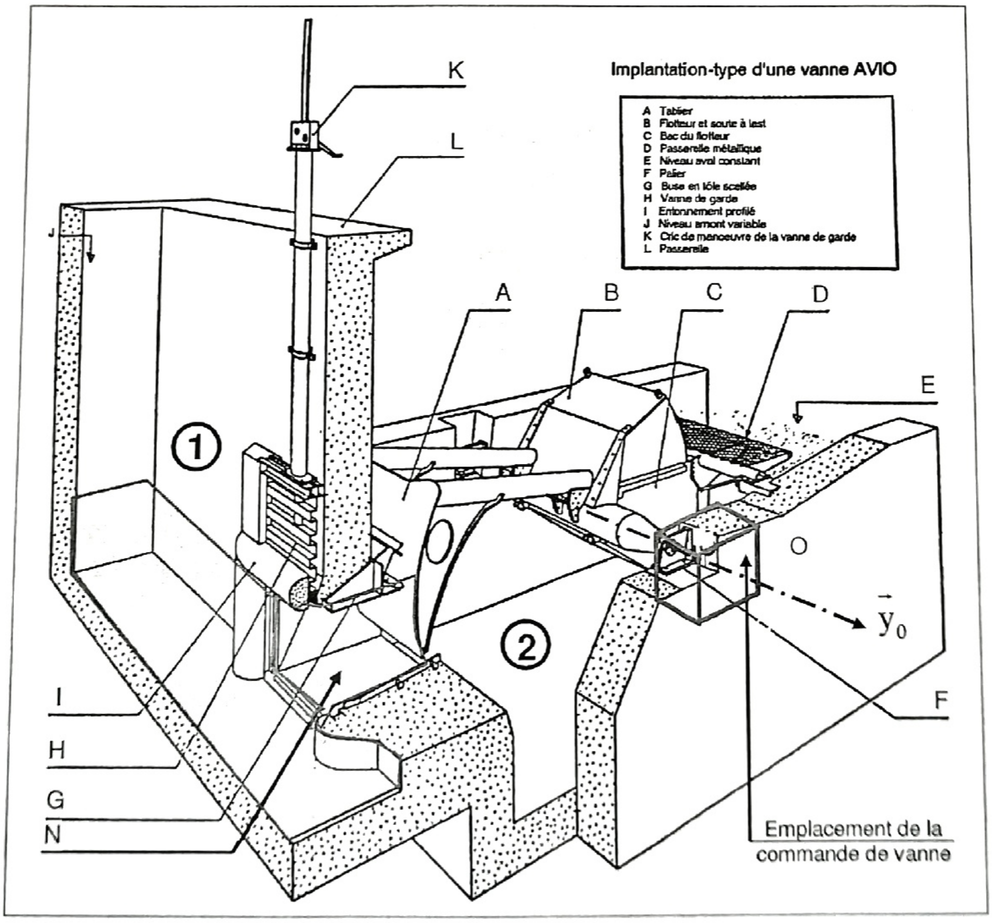


Figure 1 – Example of large capacity floodgate

The objective is to analyse the mechanism controlling the position of deck A using **the two-dimensional kinematic model** shown in Figure 2. The system comprises three solids (Figure 2):

- **the ground 0**, whose physical coordinate system is $R_0(O,\vec{x}_0,\vec{y}_0,\vec{z}_0)$.
- **piston 1**, connected to the ground **0** by a linear annular joint of axis $(A,\vec{x}_0)$ and a linear annular joint of axis $(B,\vec{x}_0)$.
- **the arm + circular floodgate deck of centre O' (solid 2)**, whose physical coordinate system is $R_2(O,\vec{x}_2,\vec{y}_{0,2},\vec{z}_2)$, connected a) to the ground **0** by a revolute joint of axis $(O,\vec{y}_0)$ and, b) to piston **1** by a linear annular joint of axis $(B,\vec{x}_2)$.

OPERATING PRINCIPLE

A pressurised fluid is inserted in the piston chamber of annular cross-section (inner radius R_i and outer radius R_e). The constant (uniform) pressure exerted by the fluid on piston 1 pushes it in the $\vec{x}_0$ direction, which induces a rocking motion of arm 2 about axis $(O,\vec{y}_0)$, thus leading to the closure of the floodgate.

HYPOTHESES

It is assumed that:
- the floodgate is stationary, closed and in the equilibrium position shown in Figure 2
- all the links are perfect.
- the mechanical action of water on deck A of the floodgate is characterised by a distribution of elementary forces in the direction $\vec{n}$, normal to the deck wall, of the form:

$$d\vec{F}(M) = -\rho g H \vec{n}\, dS$$

It is therefore supposed that the hydrostatic pressure is constant and equal to that at an average depth H. The canal is of square cross-section a*a with a << H; thus making it possible to neglect the hydrostatic pressure variation with the depth.
The weight of the parts is neglected compared with the other mechanical actions

OBJECTIVE OF THE STUDY

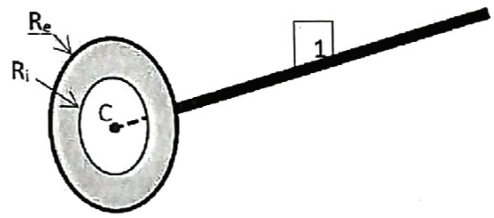
The objective is to determine the minimum pressure p exerted in the cylinder jack in order to keep the floodgate closed, along with the corresponding mechanical actions in the joints.

PART A: FORCE WRENCH (≈ 5 marks)

1- Study of the pressure forces on the piston.

The pressure p exerted on the surface of piston 1 is supposed to be uniform (constant). Determine the corresponding force wrench (sum and moment) on solid 1 at point C in terms of the pressure p, the inner and outer piston radii R_i and R_e. Explanations are required.

Hint: 3D piston geometry



2- Study of the hydrostatic forces on the floodgate deck

Determine the hydrostatic force wrench (sum and moment) on the deck of solid **2** *at point O' in terms of variables ρ, g, H, radius R, aperture a along with angles γ_1 and γ_2. Explanations are required.*

Indication: Integrate by using infinitesimal strips of width a (canal dimension) and parameter γ.

PART B: STATIC ANALYSIS : PRESSURE ON PISTON (≈ 11 marks)

Reminder: a planar model is considered.

In order to render this part independent of the previous results, the following expressions will be employed regardless of the results obtained in Part A:

The mechanical actions on piston **1**, supposed to be constant but unknown, is characterised by the wrench:

$$\{F_{(Fluid\to 1)}\}:\begin{Bmatrix}\vec{F}_{Fl/1}=F_{FL}\,\vec{x}_0\\ \vec{0}\end{Bmatrix}_C$$ where F_{FL} is **unknown.**

The hydrostatic pressure force wrench on the deck of **2**, is supposed to be known and reads:

$$\{F_{(water\to 2)}\}:\begin{Bmatrix}\vec{F}_{water/2}=X_{e2}\,\vec{x}_0+Z_{e2}\,\vec{z}_0\\ \vec{M}_{water/2}(O)=C_V\,\vec{y}_{0,1}\end{Bmatrix}_O$$ where X_{e2}, Z_{e2} and C_V are **known**.

3- Isolate solid 1

- *Apply the equilibrium conditions to solid **1** at point B. Develop the corresponding scalar equations.*

4- Isolate solid 2

- *Apply the equilibrium conditions to solid **2** at point O. Develop the corresponding scalar equations*

5- *Solve the equations.*

- *Derive the expressions of the mechanical actions exerted in the joints and on the piston in terms of the geometrical parameters along with the forces and moments generated by the hydrostatic pressure forces on solid **2**.*

6- Numerical applications

Using the numerical data in Figure 2, determine the mechanical actions in the joints.

Determine the pressure required to keep the floodgate closed in this configuration.

PART C: GRAPHICAL STATICS *(TO BE HANDED IN WITH YOUR PAPER!)* (≈ 4 marks)

7- The hydrostatic force $\vec{F}_{water/2}$ is given in Figure 3 (to be handed in), isolate solid **2** and determine by graphical construction the mechanical action from the ground **0** on solid **2** and that of piston **1** on solid **2**. Give their **numerical values** using the scale given in Figure 3. Justify your developments.

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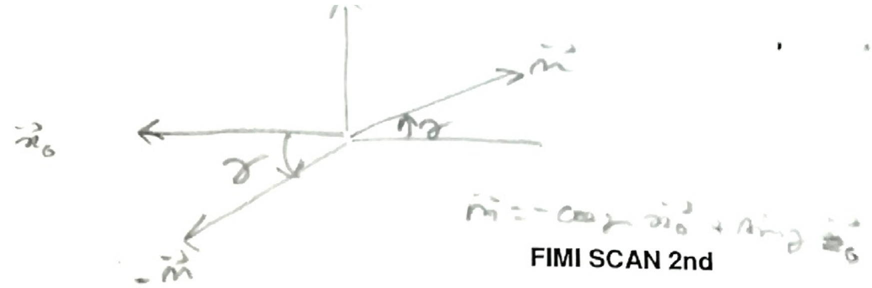

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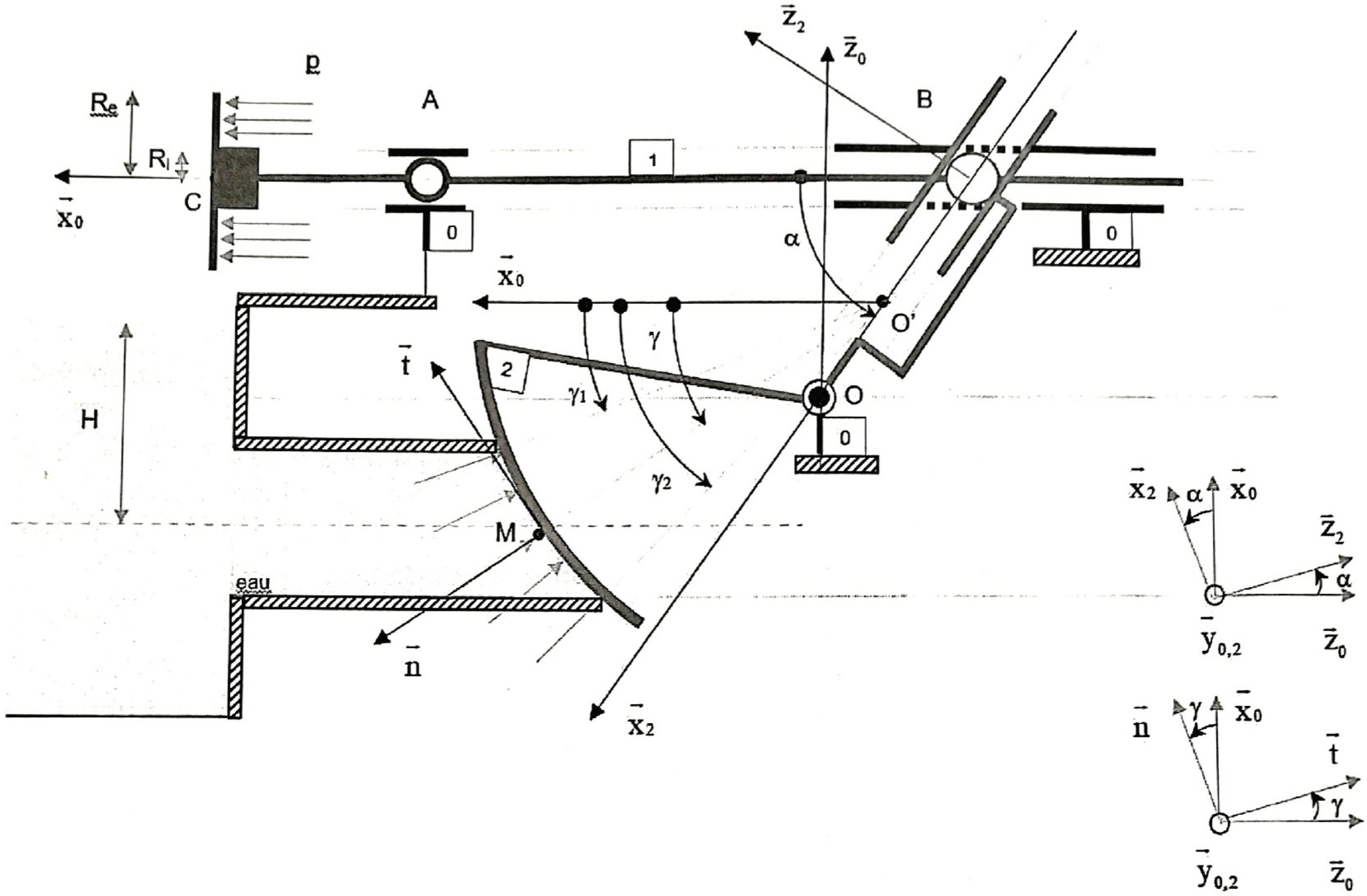

Figure 2 – Equivalent planar model

Geometrical data

Piston **1** of annular section, inner radius R_i and outer radius R_e

Floodgate deck **2** of radius R

Canal of square cross-section a*a

$\left\|\overrightarrow{AB}\right\| = \ell$; $\left\|\overrightarrow{OB}\right\| = f$

$\overrightarrow{O'M} = R\,\vec{n}$; $\left\|\overrightarrow{OO'}\right\| = e$

Numerical data

$\rho = 1000\,kg/m^3$; g = 9.81 m/s² ; H = 4 m

R_e = 0.12 m ; R_i = 0.04 m ; a = 0.2 m ; $\ell = 1.2$ m ; f = 0.6 m ; R = 0.80 m ; e = 0.25 m

X_{e2} = - 2150 N ; Z_{e2} = + 1650 N

C_V = - 200 Nm

$\gamma_1 = 25°$; $\gamma_2 = 50°$;

$\alpha = 55°$