INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

Wednesday 15th April 2020

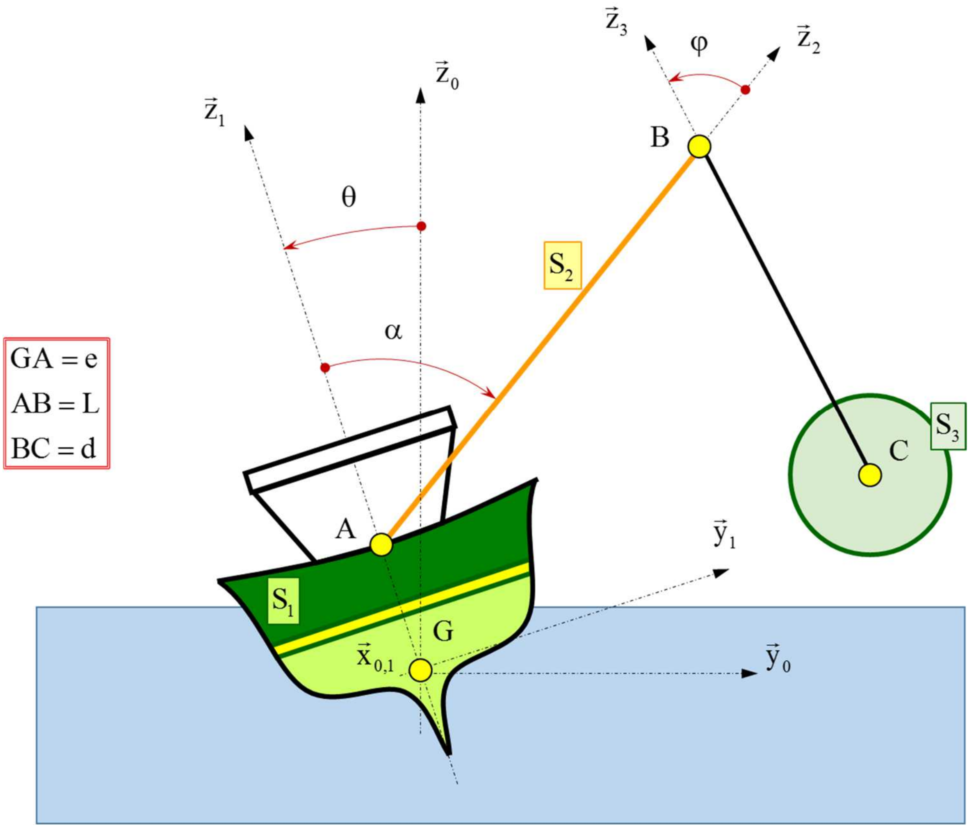
Scan 2nd - Mechanics – TEST 4

The **planar(2D)** model below is used to study the rolling of a ship carrying a load by a crane such as the seaweed harvester (*goémonier)* shown in the photo.

Seaweed harvester in operation. © Yves Gladu

It comprises:

- the ship S_1 whose motion with respect to R_0 (Galilean) is assimilated to a rotation of axis $(G, \overrightarrow{x_{0,1}})$
  Motion parameter 1/0 : θ
- arm S_2 connected to S1 by a revolute joint of axis $(A, \overrightarrow{x_{1,2}})$
  Motion parameter 2/1 : α
- of the seaweed load S_3 connected S2 by a revolute joint of axis $(B, \overrightarrow{x_{2,3}})$
  Motion parameter 3/2 : φ



<u>Mass geometry:</u>

S_1: centre of mass G, mass m_1 and moment of inertia A_1 with respect to $(G, \overrightarrow{x_1})$ which is a principal direction of inertia

S_2: centre of mass G_2 (at mid-length of AB), mass m_2 and negligible cross-section.

S_3: made of a rigid, massless link BC plus a sphere of centre C, radius R and mass m_3

External mechanical actions:

- The swell exerts a periodic torque (moment) $\overrightarrow{M_{h/1}(G)} = M_h \sin \omega t\, \overrightarrow{x_{0,1}}$ on S_1
- The **pressure of the water on the shell** combined with **the weight of the ship plus that of the arm and seaweed** on $\{S_1 + S_2 + S_3\}$ generate the following force wrench:

$$\{\overrightarrow{F_{R/1}}\} = \begin{cases} \overrightarrow{F_{R/1}} = \vec{0} \\ \overrightarrow{M_{R/1}}(G) = M_R \overrightarrow{x_{0,1}} \end{cases}$$

For the sake of simplicity, it is further assumed that the moment varies linearly with the angle of rolling θ such that $M_R = -K\theta$.

Part I – Mass geometry :

Q 1 - Give the matrix of inertia of S_3 at point C and then at point B (in terms of m_3, R and d), identify the moment of inertia I_3 of S_3 with respect to axis $(B, \overrightarrow{x_3})$.

Q 2 - Give the matrix of inertia of S_2 at point A (in terms of m_2 and L).

Q 3 - Assuming that $\alpha = cst$ (i.e. S_2 does not move with respect to S_1), give the moment of inertia I_Σ of $\Sigma = \{S_1 + S_2\}$ with respect to axis $(G, \overrightarrow{x_1})$.

Part II – Kinetics :

Considering the following hypotheses / simplifications:

- $\alpha = Cste$ S_2 does not move with respect to S_1
- $e = 0$ which implies that $A \equiv G$
- G is fixed in the Galilean frame R_0
- the mass of S_2 is neglected compared with that of S_1
- S_3 is assimilated to a point (lumped) mass m_3 located at point C

Q 4 - Calculate the sum and moment at G of the Galilean dynamic wrench for $\Sigma = \{S_1 + S_2\}$.

Q 5 - Calculate the sum and moment at B of the Galilean momentum (kinetic) wrench for S_3.

Hint: keep the expressions of the components of $\vec{V}(C/0)$ in R_2 and R_3

Q 6 - Calculate the sum and moment at B of the Galilean dynamic wrench for S_3.

Hint: keep the expressions of the components of $\vec{A}(C/0)$ in R_2 and R_3

Part III – Dynamics :

Using the same hypotheses and simplifications as in Part II

Q 7 - Develop the dynamic moment theorem for $\Sigma_1 = \{S_3\}$ projected on $(B, \overrightarrow{x_{3,2}})$.

Develop the dynamic moment theorem for $\Sigma_2 = \{S_1 + S_2 + S_3\}$ projected on $(G, \overrightarrow{x_{0,1}})$.

Indication: do not try to express the dynamic moment of S_3 at point G but use the notation $\overrightarrow{\delta_3^0(G)} = \delta_3 \overrightarrow{x_{0,1}}$.