MECHANICS – Test 1

16th November 2020 _ 1:30 hour (10:15-11:45)

Authorised material: personal formula sheet (1 page + 1 formula sheet for the joints)

Non-programmable pocket calculator

Write and sign on your paper the following sentence: **"I certify that I will not cheat, and I will follow the guidelines given in the subject, I will not chat with anyone else than the teacher supervising the exam"**.

Particular attention will be paid regarding the quality of writing and the justifications.

The two exercises are independent.

1 Aerodynamic actions on a helicopter main rotor (~12pts)

The main rotor of a helicopter is often made of three blades positioned 120° apart. One of these blades is represented in Figure 1, Solid S_2, linked to the helicopter (solid S_1) by a revolute joint of axis $(O, \vec{z}_{1,2})$ and parameter θ (S_2/S_1), where $\theta = (\vec{x}_1, \vec{x}_2)$.

Motion of the blade:

The motion of the blade S_2 with respect to S_1 is a rotation about axis $(O, \overrightarrow{z_{1,2}})$. Considering a point M belonging to the blade longitudinal axis $(\overrightarrow{OM} = r\,\overrightarrow{x_2})$, its linear velocity with respect to R_1 is given by:

$$\vec{V}(M/1) = r\,\dot{\theta}\,\vec{y}_2 = r\,\omega\,\vec{y}_2$$

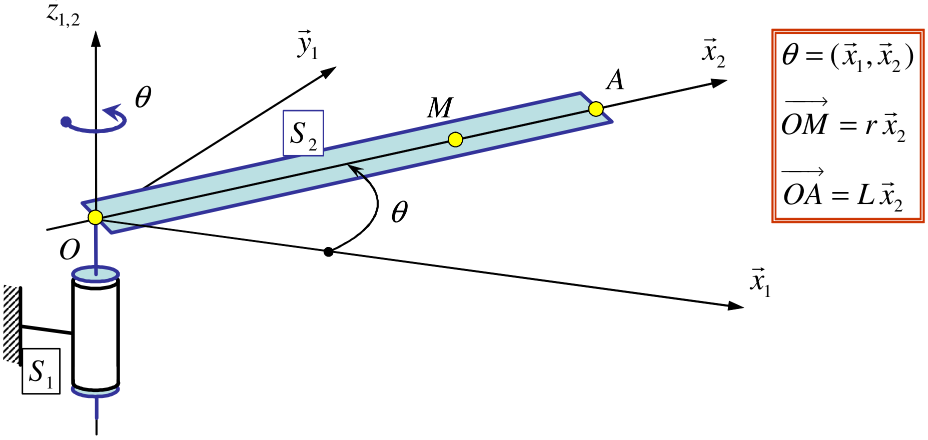

Figure 1: Blade of a helicopter main rotor

Aerodynamic forces on the blade: Figure 2 shows the distributions of the aerodynamic forces along a blade of length L. At every elemental segment of length *dr* centred at M, the corresponding elemental aerodynamic forces are expressed as:

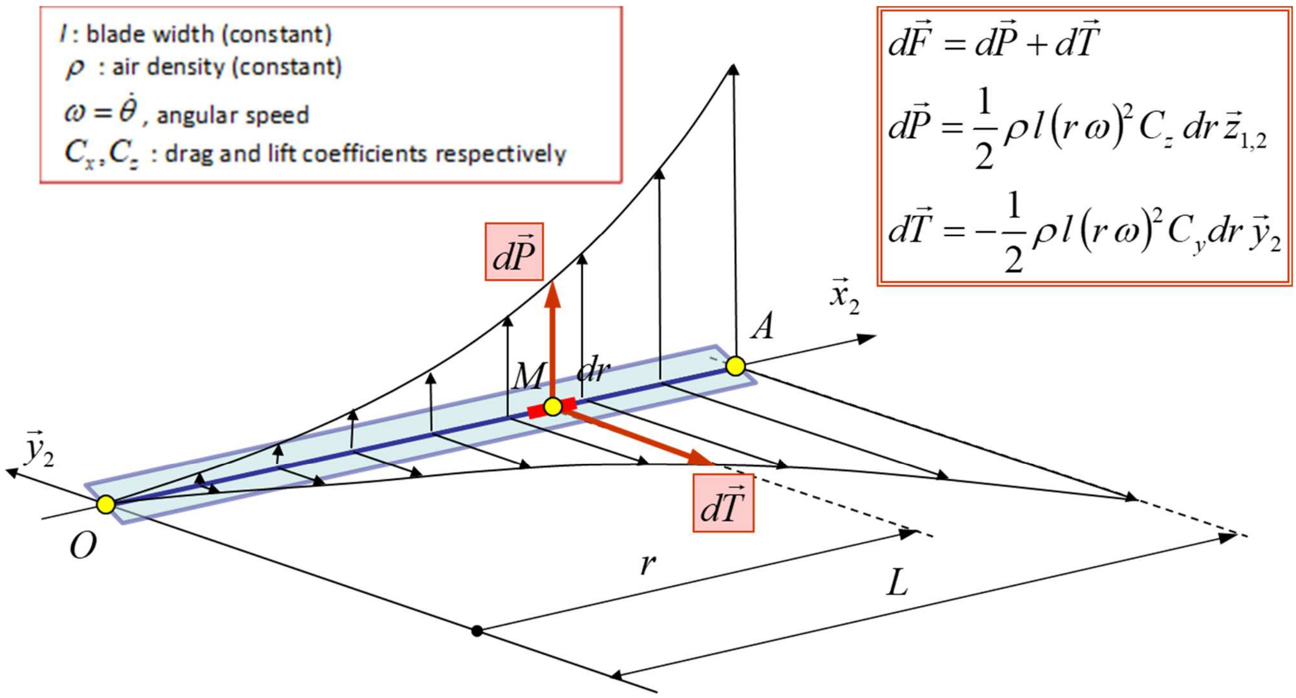

Figure 2: Aerodynamic forces on the blade

Where $d\vec{P}, d\vec{T}$ are the elemental **lift** and **drag** forces respectively.

1.1 - Find the sum and moment at O of the aerodynamic forces wrench on the blade S_2.

1.2 - Assuming that these mechanical actions do not vary with the rotor rotation, give the expressions of the sum and moment at O of **the aerodynamic wrench generated by the three blades of the main rotor (three blades are 120° apart)**.

** It will be admitted with no proof that:*

- the drag forces cancel two by two (resisting force).

- the moment generated at point O by the lift forces $d\vec{P}$ is nil.

Global mechanical actions on the helicopter

When flying in translation and at constant speed, the helicopter shown in Figure 3 is submitted to:

- Its weight (mass m, centre of mass G, $\overrightarrow{z_1}$ vertical upward),
- The aerodynamic actions on the main and tail rotors are characterised by the two force wrenches :

$$\{T_{Aero/MainRotor}\}_O = \begin{cases} \overrightarrow{R_{A/MR}} = R_x.\overrightarrow{x_1} + R_y.\overrightarrow{y_1} + R_z.\overrightarrow{z_1} \\ \overrightarrow{M_{A/MR}(O)} = M_R.\overrightarrow{z_1} \end{cases} \qquad \{T_{Aero/TailRotor}\}_B = \begin{cases} \overrightarrow{R_{A/TR}} = Q.\overrightarrow{y_1} \\ \overrightarrow{M_{A/TR}(B)} = M_Q.\overrightarrow{z_1} \end{cases}$$

- The aerodynamic actions on the cabin represented by the wrench :

$$\{T_{Aero/Cabin}\}_A = \begin{cases} \overrightarrow{R_{A/Cb}} = T_1.\overrightarrow{x_1} + T_2.\overrightarrow{y_1} + P_C.\overrightarrow{z_1} \\ \overrightarrow{M_{A/Cb}(A)} = \vec{0} \end{cases}$$

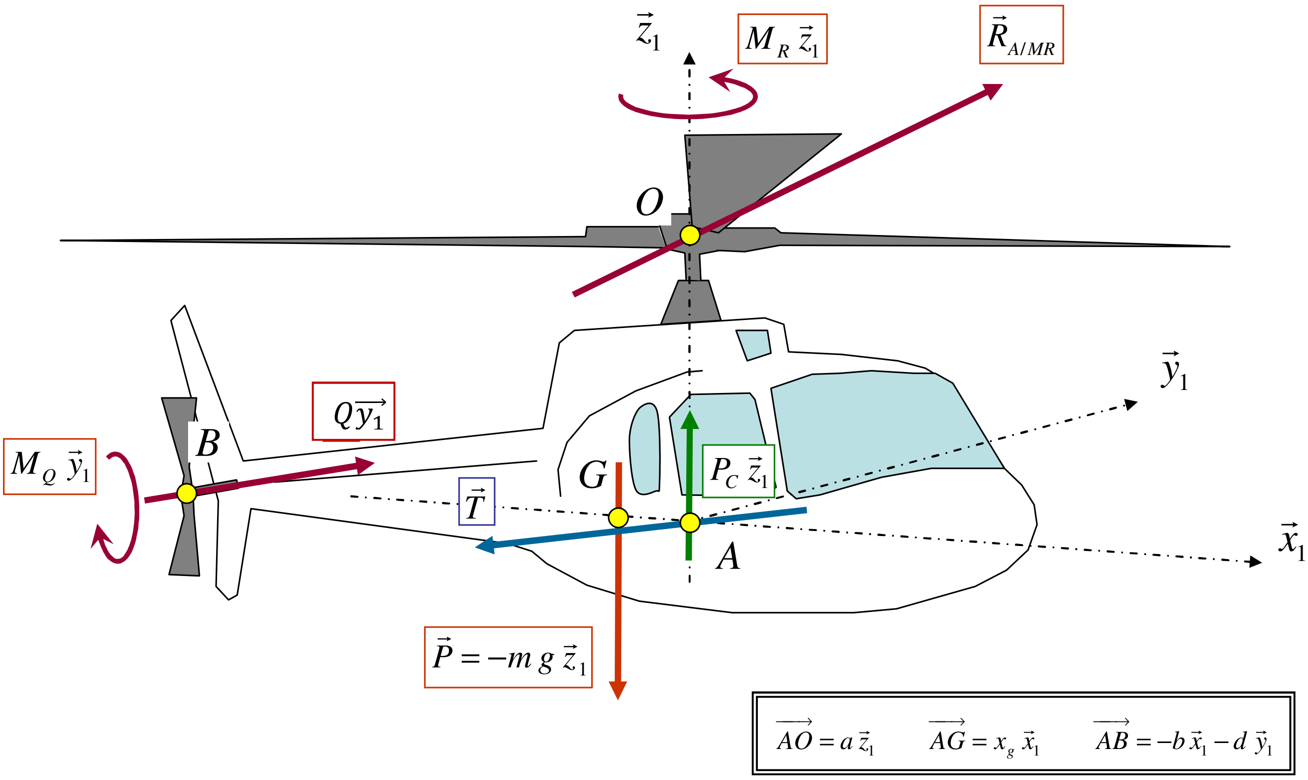

Figure 3: Mechanical actions on the helicopter

1.3 - Develop the equations of equilibrium for the helicopter and deduce the practical interest of the tail rotor.

***** that question is independent from the previous ones***

1.4 - Specify the particular motion of the helicopter associated with the aerodynamic wrench used in **I.2** .

2 Graphical Statics : Study of a chairlift disengageable clamp (~8pts)

The proposed study concerns a detachable chairlift clamp. Chairlift transport is usually achieved by a moving cable to which nacelles are attached. In order to have a higher ascent rate, the cable running speed must be increased, possibly leading to safety problems when boarding and disembarking skiers. A device is therefore used which allows the chair to be disengaged from the cable in the embarkation and disembarkation stations.

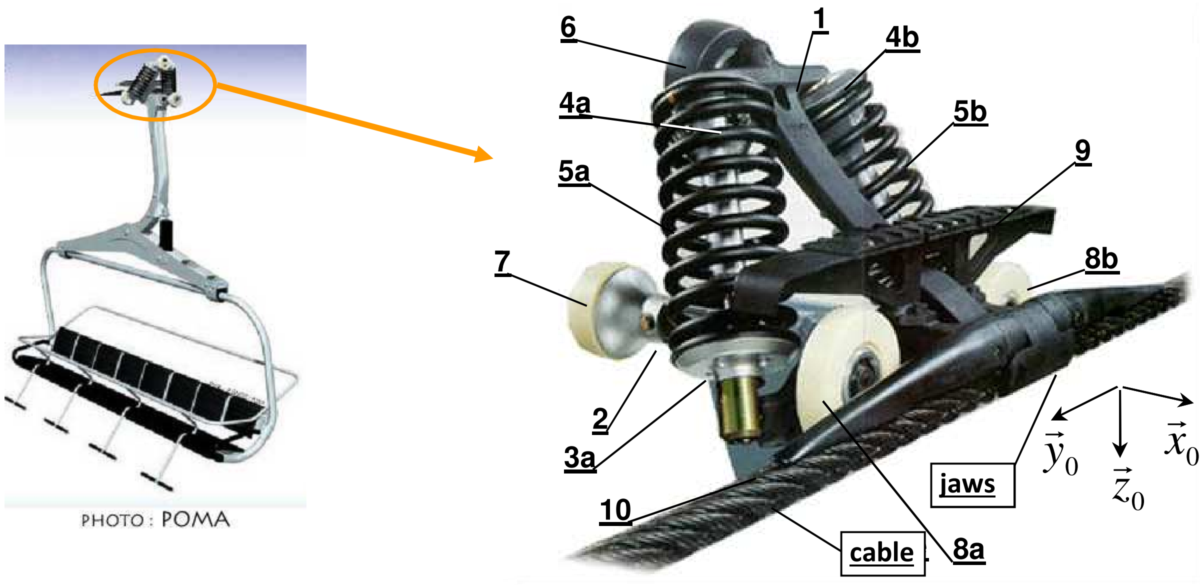

Figure 4: Chairlift disengageable clamp

The clamping system (Figure 4) makes disengagement possible. It is symmetrical with respect to the plane (jaw centre (A), $\vec{x_0}, \vec{z_0}$) with two halves labelled a and b respectively. On account of this symmetry, an equivalent planar model can be constructed which is shown in Figure 5 for the passage-in-station mode. In this context, rollers (6, 7, 8a and 8b) are in contact with specific rails (attached to the ground) imposing their positions. The clamp is open and has no contact with the cable.

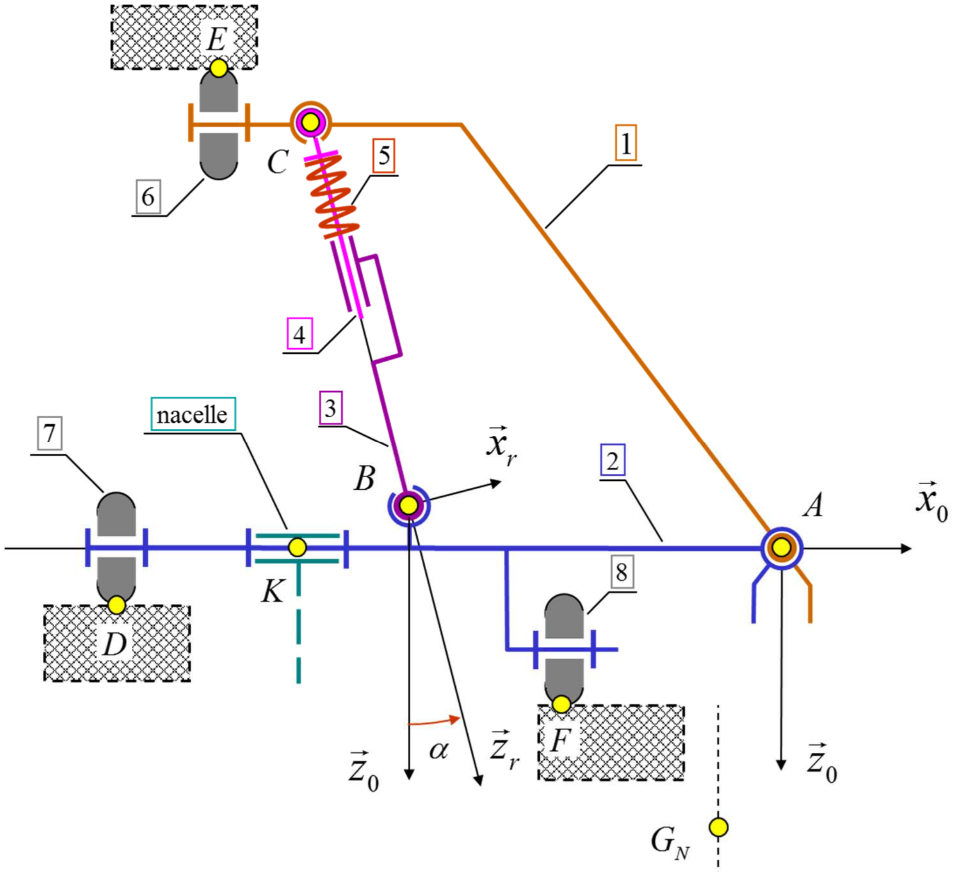

Figure 5: Planar model of a chairlift disengageable clamp

Disengageable clamp in passage-in-station mode

The **graphical analysis** will be developed using the figure in the last page. The figure is represented with a background grid. To ease the drawing on your personal paper sheet, the points coordinates are given hereafter in the basis $\left(K, \vec{z}_0, \vec{x}_0\right)$:

$K(0, 0) \quad A(0, 15) \quad B(-1.5, 3.5) \quad C(-13.5, 0.5) \quad D(2, -5.5) \quad E(-15, -2.5) \quad F(5, 8.5)$

The following hypotheses will be used:

- Action $\vec{R}_{Nacelle/2}$ from the nacelle on part 2 is known it is a sliding vector passing through A,
- Action $\vec{R}_{4/1}$ developed by the spring is known,
- The contact at E between the rail and roller 6 is frictionless so that the corresponding mechanical action is purely vertical along $\vec{z}_0$.

Drawing scale: **2083 N/cm**

- $\vec{R}_{4/1}$ is represented by a vector of length 6 cm,
- $\vec{R}_{Nacelle/2}$ is represented by a vector of length 3,8 cm.

***Remark :** one will justify all the graphical constructions in the answer sheet*

2.1 – Using the equilibrium of the assembly {1,6} draw the actions $\vec{R}_{Rail/6}$and $\vec{R}_{2/1}$ and give their magnitude in Newton.

2.2 – Using the equilibrium of the assembly {2, 7, 8} (*NB: treat the following questions in order*)

A – Build the resulting force $\vec{R}_{(Nacelle+1)/2}$ and give its magnitude in Newton
Remark: this resultant is the sliding vector passing through A accounting for the combined actions of the nacelle and solid 1 on solid 2.

B - Build the resulting force $\vec{R}_{(Nacelle+1+3)/2}$ and give its magnitude in Newton
Remark : this resultant is the sliding vector whose line of action will be determined, accounting for the combined actions of the nacelle, solid 1 and the spring system on solid 2

C – Why the orientation of this resulting force is interesting from a functional viewpoint? Explain qualitatively the methodology to be put in place in order to solve the equilibrium of the assembly {2, 7, 8} NB: do not develop the graphical construction

LAST NAME :

First Name :

Group :