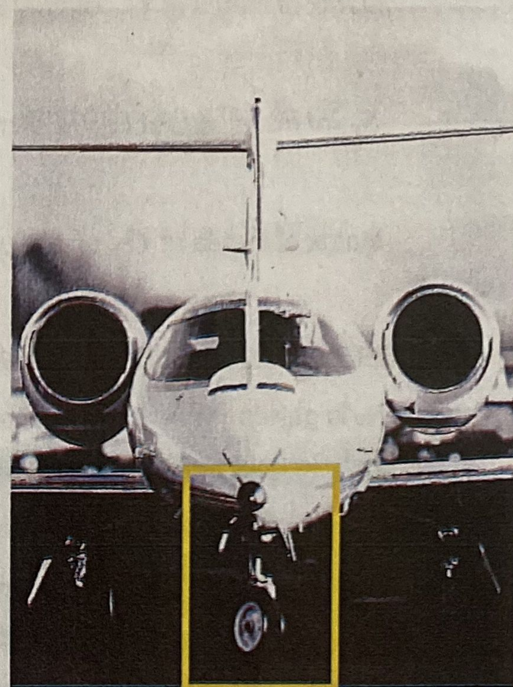
INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

Monday 13th June 2022

Final test of Mechanics of Systems - Shimmy of an aircraft wheel

Duration : 3 hours

Authorised : Calculators + 2 personal formula sheets + table of usual joints + table of inertia matrices.



1 - Presentation - Modelization :

The proposed study focuses on the shimmy motion (vibration) of the wheel of a front landing gear (framed in the figure on the right). A video of this behaviour can be viewed (after the test !) at the link : https://www.youtube.com/watch?v=-ElG5pW0y4U

More generally, this phenomenon is likely to occur on all wheels mounted on a « rotating » and deformable support (trailers, motorbikes, shopping trolley, etc...).

This study proposes to highlight the shimmy on a simplified mechanical model, presented in **figure 1**.

In addition to the solid S_0 which represents the runway track, this mechanism is composed of four solids : S_{1^*}, S_1, S_2 and S_3.

- S_{1^*} represents the aircraft, whose **motion is known** and consequently won't be studied. This motion is described as a translation, which is linear (straight) and uniform (with constant velocity v) along the direction $\vec{x}_0$.
- S_1 represents the landing gear leg. This leg is a deformable structure, whose extremity carries the fork S_2. In the proposed model, only the lateral motion, induced by the elastic deformation of the leg, is taken into account. The link S_1 / S_{1^*} is thus defined as a prismatic joint of axis $(O_1, \vec{y}_{0,1})$.

  Parameter of the motion 1/1 :* $y = \overrightarrow{O_1^* O_1} . \vec{y}_{0,1}$

- S_2 represents the fork which carries the wheel S_3 ; S_2 is linked to S_1 by a revolute joint of axis $(O_1, \vec{z}_{0,1,2})$

  Parameter of the motion 2/1 : $\theta = (\vec{x}_{0,1}, \vec{x}_2)$

- S_3 is the wheel of the landing gear ; S_3 is linked to the fork S_2 by a revolute joint of axis $(O_3, \vec{y}_{2,3})$

  Parameter of the motion 3/2 : $\varphi = (\vec{x}_2, \vec{x}_3)$

- Furthermore, S_3 is in contact with the runway track at I.

<u>Mass geometry :</u>

- S_1 is considered as a solid **of negligible mass**,
- S_2 is a solid, of mass m_2, of centre of mass G_2 such that $\overrightarrow{O_1G_2} = -a\,\vec{x}_2$

Matrix of inertia at G_2 of S_2 : $\quad \overline{\overline{I}}(G_2,S_2) = \begin{pmatrix} A_2 & 0 & -E_2 \\ 0 & B_2 & 0 \\ -E_2 & 0 & C_2 \end{pmatrix}_2$

- S_3, of mass m_3, of centre of mass O_3, is a part of revolution of axis $(O_3,\vec{y}_{2,3})$.

Matrix of inertia at O_3 of S_3 : $\quad \overline{\overline{I}}(O_3,S_3) = \begin{pmatrix} A_3 & 0 & 0 \\ 0 & B_3 & 0 \\ 0 & 0 & A_3 \end{pmatrix}_3$

<u>Assumptions and specific mechanical actions :</u>

- Ro is galilean with $\vec{z}_0$ vertical and ascending.
- Except the contact between the wheel and the runway track, all the joints are perfect.
- The geometric data are systematically given and also repeated in figure 1.
- A traction / compression spring, linear and massless, of stiffness k, whose action is nil when y is nil, is mounted between S_1 and S_{1*}. This spring models the elastic transversal deformation of the landing gear leg. For convenience, it has been represented slightly shifted, but its line of action is the line $(O_1,\vec{y}_{0,1})$.
- A torsional "anti-shimmy" damper, of axis $(O_1,\vec{z}_{0,1,2})$ and viscous damping coefficient b, not represented in figure 1, is mounted between S_1 and S_2.
- The wrench of mechanical actions of the runway track on the wheel is given by :

$$\{T_{0/3}\} = \begin{cases} \vec{R}_{0/3} = X_{03}\,\vec{x}_2 + Y_{03}\,\vec{y}_2 + Z_{03}\,\vec{z}_2 \\ \vec{M}_{0/3}(I) = N_{03}\,\vec{z}_2 \end{cases}$$

  - The transverse effort Y_{03} is given by the drift law of the tyre :

$$Y_{03} = -D\,\delta \qquad \text{where } \delta = \arctan\left(\frac{\vec{V}(I/0).\vec{y}_2}{\vec{V}(I/0).\vec{x}_2}\right) \text{ is the drift angle of the tyre}$$

  - N_{03} is the self aligning moment of the wheel, given by : $N_{03} = A_0\,\delta$
  - D and A_0 are **known constants**, relative to the tyre's behaviour.
  - **The normal effort Z_{03} will be considered as known,** determined using another model (not studied here) representing the global dynamic behaviour of the aircraft.
- The conditions for the contact tyre / track impose the longitudinal non-slipping of the wheel on the track, which results in :

$$\vec{V}(I,3/0).\vec{x}_2 = 0$$

2 - Kinematics ~ 3 Pts

Q 1 - Draw the graph of links.

Q 2 - Write the equation translating the condition of contact at I. Explain why it constitutes a so-called "mounting" equation and explain why it won't be counted in the complete table unknowns / equations.

Q 3 - Write the constraint equation resulting from the longitudinal non-slippping at I.

Q 4 - Express the drift angle δ depending on the motion parameters.

3 - Kinetics ~ 5 Pts

Q 5 - Determine the galilean dynamic wrench of S_1 at O_1.

Q 6 - Determine the galilean kinetic wrench of S_2 at G_2.

Q 7 - Determine the galilean dynamic wrench of S_2 at G_2 and then at O_1.

Q 8 - Determine the galilean dynamic wrench of S_3 at O_3 and then at O_1.

4 - Dynamics ~ 8 Pts

Q 9 - Establish the complete table of unknowns / equations, and then determine the reduced table (i.e. minimum system), which permits the obtention of the motion equations.

Q 10 - Develop the equations of the minimum system.

Q 11 - Verify that the position defined by $y = y_S = 0$, $\theta = \theta_S = 0$ and $\dot{\varphi} = \omega_S = Cst$ is an admissible stationary position of the mechanism. Determine ω_S and the contact mechanical actions $X_{03\,S}$, $Y_{03\,S}$ and $N_{03\,S}$ in this context.

Q 12 - The small motions of the system will now be analyzed, by writing : $y = y_S + \overline{y} = \overline{y}$ and $\theta = \theta_S + \overline{\theta} = \overline{\theta}$, with $\overline{y}$, $\overline{\theta}$ and their time-derivatives which are considered small.

- Show, by linearizing the equation translating the longitudinal non-slipping at I, that the assumption of a plane's motion at speed $v = Cst$ leads to its own rotation velocity of the wheel $\dot{\varphi} = \omega_S = Cst$.
- After having linearized the expression of the drift angle δ, give the expressions of X_{03}, Y_{03} and N_{03} in the context of small motions.
- Write the system of equations of small motions in $\overline{y}$ and $\overline{\theta}$, considering that $A_0 << \ell D$.

  Expressing : $I_\Delta = C_2 + A_3 + m_2 a^2 + m_3 \ell^2$ and $M_\Delta = m_3 \ell + m_2 a$

  the system will be written under the matrix form (2*2) :

$$[M]\ddot{X} + [C]\dot{X} + [K]X = 0 \quad \text{where} \quad X = \begin{pmatrix} \overline{\theta} \\ \overline{y} \end{pmatrix}$$

Remark : The search for the time-dependent solutions of this system, **which is not requested**, would demonstrate that the motions become unstable if $v < 0$ or $k\ell - D < 0$.

5 - *Energetics* ~ 4 Pts

Q 13 - Compute the galilean power developed by the mechanical actions applied to the isolated system $\Sigma = \{S_1 + S_2 + S_3\}$.

Remark : Be careful to distinguish the galilean power developed by the external mechanical actions from the power developed by the internal actions.

Q 14 - Determine the galilean kinetic energy of $\Sigma = \{S_1 + S_2 + S_3\}$.

$$\overrightarrow{O_1^*O_1} \cdot \vec{z}_{0,1} = d$$
$$\overrightarrow{O_1O_3} = -\ell\,\vec{x}_2$$
$$\overrightarrow{IO_3} = R\,\vec{z}_{0,1}$$
$$\overrightarrow{O_1G_2} = -a\,\vec{x}_2$$

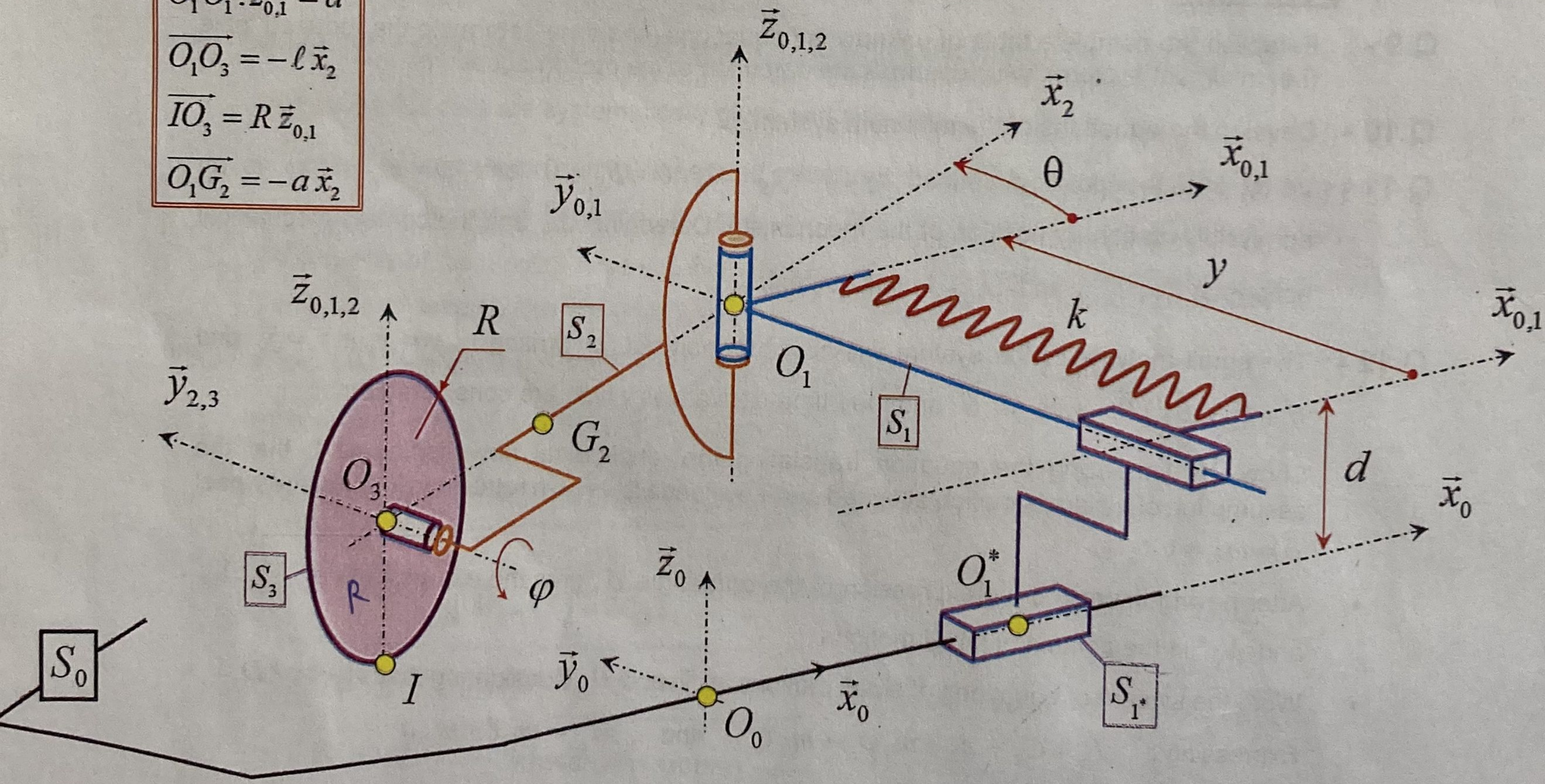


Figure 1 : Scheme of the lower part of the landing gear.