Mechanics of Systems – Test n°3

Monday 11th april 2022 – 1h30

Authorised : Calculators and Personal formula sheets (3 pages + table of usual joints + table of inertia matrices).

Study of a pinball strike mechanism

Indicative scale : Part 1 (7 pts), Part 2 (13 pts)

Presentation of the system

The system under study is the « strike mechanism » of a pinball machine (**Figure 1**). Two of these electromechanical assemblies are present in a pinball machine, and permit a player to return the ball to the top of the board, to aim at targets and corridors in order to score maximum points.

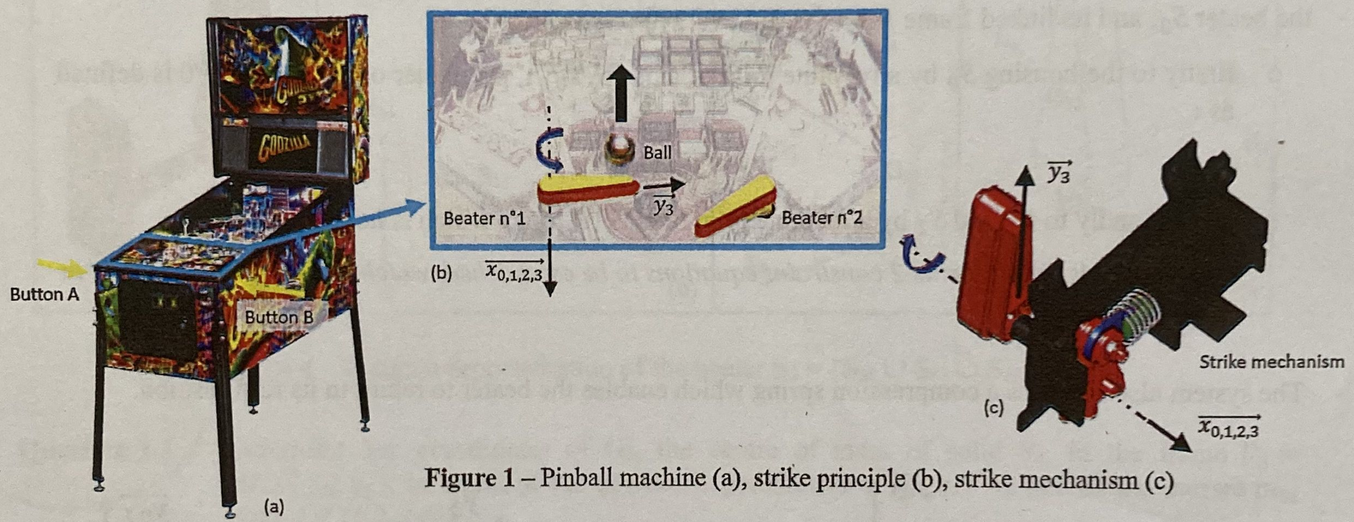


Figure 1 – Pinball machine (a), strike principle (b), strike mechanism (c)

The system's actuator is an electro-magnet, which consists of an electrical coil and a plunger core (**Figure 2**). When the player pushes the button A (**Figure 1a**), the coil is powered. Because of the electromagnetic coupling, the plunger core is attracted into the actuator's body (**Figure 2b**). This induces the rotation of the beater through a connecting rod. The beater thus « strikes » the ball. Once the button A is released, the power goes off and the plunger core is pushed back to its initial position, by a compression spring.

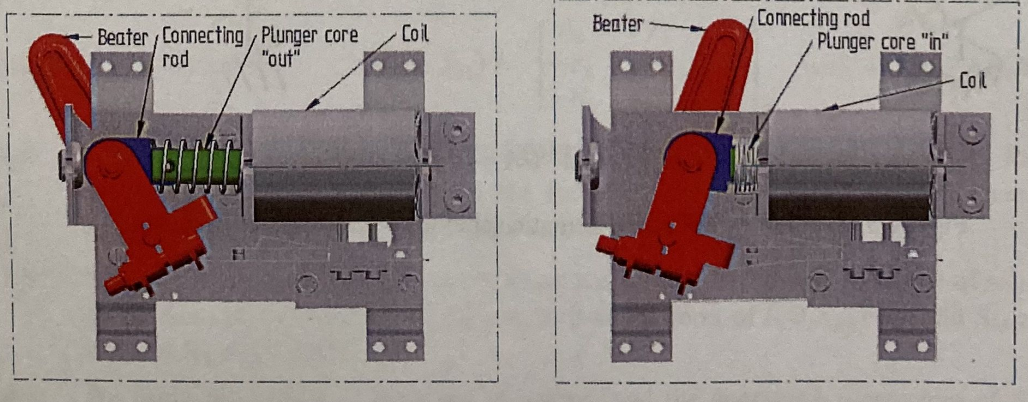


Figure 2 – Positions of the system : **button A released** (a) and **button A pressed** (b)

Parametrization :

The strike mechanism is modeled by the planar and spatial kinematic schemes in **figure 3.** Thereafter, the hypothesis of **planar problem** will be assumed for this problem.

The model is composed of :

- the pinball machine's housing S_0, fixed to the ground, and its linked frame $R_0 = (O, \vec{x_0}, \vec{y_0}, \vec{z_0})$
- the plunger core S_1, and its linked frame $R_1 = (B, \vec{x_1}, \vec{y_1}, \vec{z_1})$; S_1 is linked to the housing S_0 by a prismatic joint of direction $(A, \vec{z_{0,1}})$; parameter of the motion 1/0 is defined as :

$$z = \vec{AB}.\vec{z_{0,1}}$$

- the connecting rod S_2, and its linked frame $R_2 = (B, \vec{x_{1,2}}, \vec{y_2}, \vec{z_2})$; S_2 is linked to the plunger core S_1 by a revolute joint of axis $(B, \vec{x_{1,2}})$; parameter of the motion 2/1 is defined as :

$$\theta = (\vec{y_1}, \vec{y_2})$$

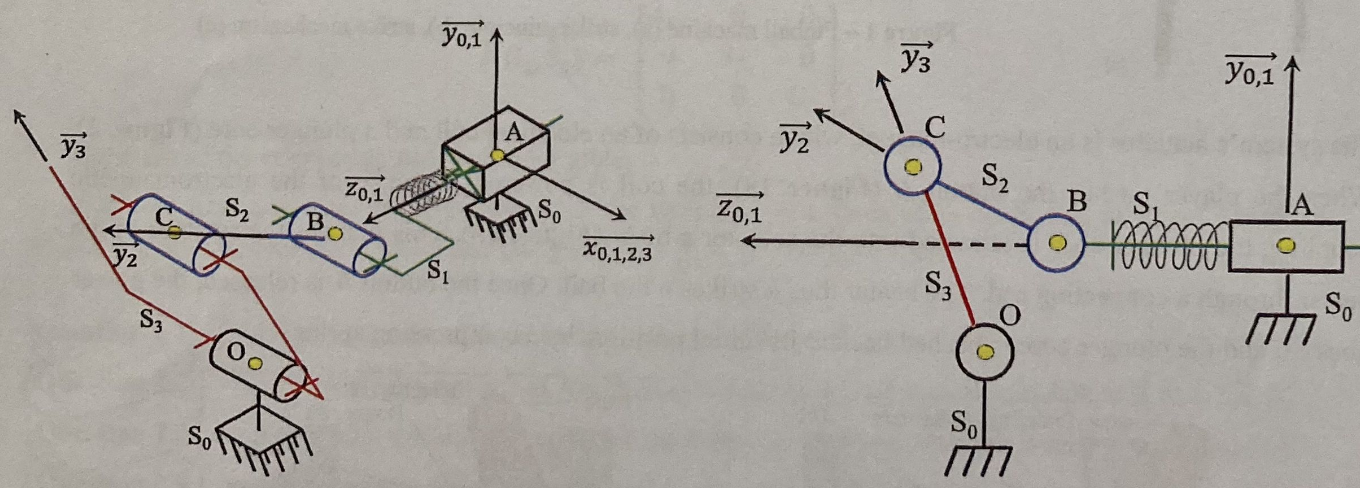
- the beater S_3, and its linked frame $R_3 = (O, \vec{x_{2,3}}, \vec{y_3}, \vec{z_3})$; S_3 is linked :
  - firstly to the housing S_0 by a revolute joint of axis $(O, \vec{x_{0,3}})$, parameter of the motion 3/0 is defined as :

$$\psi = (\vec{y_0}, \vec{y_3})$$

  - and secondly to the rod S_2 by a revolute joint of axis $(C, \vec{x_{2,3}})$, which is not parametered.

    Remark : *this joint permits 2 constraint equations to be established, which will not be determined in the present study.*

- The system also includes a compression spring which enables the beater to return to its rest position.



$\vec{OC} = e\,\vec{y_3}$ and $\vec{OA} = c\,\vec{y_0} - a\,\vec{z_0}$

Figure 3 – Spatial and planar kinematic scheme of the strike mechanism.

Part 1 : Mass geometry of solid S_3

Hypotheses :

The beater S_3 (**Figure 4**) is composed of 4 homogeneous solids S_{31}, S_{32}, S_{33} and S_{34}, rigidly bound together :

- the solid S_{31} is a half-cylinder, of mass m_{31} and has a centre of mass G_{31},
- the solid S_{32} is a triangular prism, of mass m_{32} and has a centre of mass G_{32},
- the solid S_{33} is assimilated to a full rod, of length L_{33}, mass m_{33} and has a centre of mass G_{33},
- the solid S_{34} is a rectangle parallelepiped, of mass m_{34} and has a centre of mass G_{34}.

For the whole system $S_3 = \{S_{31} \cup S_{32} \cup S_{33} \cup S_{34}\}$, the centre of mass will be called G_3 and its mass m_3.

Points G_{31}, G_{32}, G_{33} and G_{34} are located with respect to point O (**Figure 4c**).

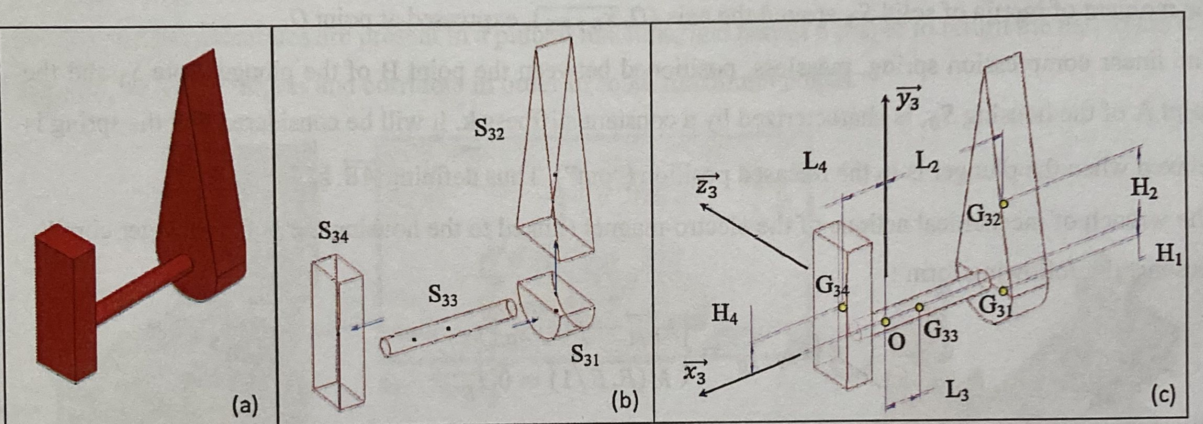


Figure 4 - Volumic representation of the beater $S_3 = \{S_{31} \cup S_{32} \cup S_{33} \cup S_{34}\}$

Question 1.1 Determine the coordinates of G_3, the centre of mass of solid S_3, in the frame $R_3 = (O, \vec{x_3}, \vec{y_3}, \vec{z_3})$, in terms of the geometric parameters of **Figure 4c** and of the masses m_{3i} (with $i \in \{1,2,3,4\}$).

Question 1.2 When the system was designed, the point O - centre of the revolute joint between S_0 and S_3 – was placed such that G_3 belongs to the plane $(O, \vec{y_3}, \vec{z_3})$.

Write the corresponding geometric condition and the associated equation.

Question 1.3 Give, without calculation, the matrix of inertia of solid S_{33}, expressed in G_{33} in the frame $R_3 = (O, \vec{x_3}, \vec{y_3}, \vec{z_3})$, in terms of L_{33} and m_{33}.

Thereafter, the matrices of inertia of the sub-solids S_{31}, S_{32} and S_{34} will be expressed at M in the frame R_3 as follows, using Binet's notation :

$$\bar{\bar{I}}(M, S_{3i}) = \begin{bmatrix} A_i & -F_i & -E_i \\ -F_i & B_i & -D_i \\ -E_i & -D_i & C_i \end{bmatrix}_{R_3} \quad \text{with } i \in \{1,2,4\}$$

Question 1.4 Give without calculation (but with justification), and using Binet's notation, the form of $\bar{\bar{I}}(G_{31}, S_{31})$ of solid S_{31} at G_{31} in the frame of $R_3 = (O, \vec{x_3}, \vec{y_3}, \vec{z_3})$, and then calculate $\bar{\bar{I}}(O, S_{31})$ of solid S_{31} at O in the same basis.

Question 1.5 Give without calculation, and using Binet's notation, the form of $\bar{\bar{I}}(O, S_{32})$ of solid S_{32} at O in the basis of $R_3 = (O, \vec{x_3}, \vec{y_3}, \vec{z_3})$, as well as the form of $\bar{\bar{I}}(O, S_{34})$ of solid S_{34} at O in the basis of $R_3 = (O, \vec{x_3}, \vec{y_3}, \vec{z_3})$.

Question 1.6 By using the results of previous questions, and the geometric parameters of **Figure 4c**, express the matrix of inertia $\bar{\bar{I}}(O, S_3)$ of solid S_3 at O in the basis of $R_3 = (O, \vec{x_3}, \vec{y_3}, \vec{z_3})$.

Part 2 : Kinetics and Dynamics

Assumptions :

- The results of part 1 remain valid,
- The problem is considered as planar,
- The frame $R_0 = (O, \overrightarrow{x_0}, \overrightarrow{y_0}, \overrightarrow{z_0})$ is galilean with axis $\overrightarrow{x_0}$ vertical,
- The action of gravity along the vertical axis $\overrightarrow{x_0}$ is neglected, except for the solid S_3,
- All joints are assumed to be perfect,
- The position of the centre of mass G_3 of solid S_3, mass m_3, is defined by $\overrightarrow{OG_3} = y_G \overrightarrow{y_3}$. One will note A_3 the moment of inertia of solid S_3 around the axis $(O, \overrightarrow{x_{0,1,2,3}})$, expressed at point O.
- The linear compression spring, massless, positioned between the point B of the plunger core S_1 and the point A of the housing S_0, is characterized by a constant stiffness **k**. It will be considered that the spring is relaxed when the plunger is in the released position ("out"). Thus defining $\overrightarrow{AB}.\overrightarrow{z_{0,1}} = z_0$.
- The wrench of mechanical actions of the electro-magnet (linked to the housing S_0) on the plunger core S_1 presents the following form :

$$\{F_{E/1}\}_B = \begin{Bmatrix} \overrightarrow{R_{E/1}} = f(z)\, \overrightarrow{z_{0,1}} \\ \overrightarrow{M(B, E/1)} = \vec{0} \end{Bmatrix}_B$$

The constitutive law, linking the effort f to the plunger's position z is considered as known.

- The plunger core S_1 is assimilated to a solid of revolution, of mass m_1, having a centre of mass G_1, such as $\overrightarrow{G_1B} = d\, \overrightarrow{z_{0,1}}$, and the matrix of inertia

$$\bar{\bar{I}}(G_1, S_1) = \begin{bmatrix} A_{S_1} & 0 & 0 \\ 0 & A_{S_1} & 0 \\ 0 & 0 & C_{S_1} \end{bmatrix}_{R_1}$$

- The mass of the connecting rod S_2 is negligible.
- It will be admitted without justification that the wrenches of actions of S_3 on S_2 and of S_1 on S_2 can be reduced to sliding vectors of action line (BC) and sum $\overrightarrow{R_{1/2}} = -\overrightarrow{R_{3/2}} = F\overrightarrow{y_2}$.

Question 2.1 Draw the graph of links and the change of basis diagrams.

Question 2.2 Determine the wrench of mechanical actions of the compression spring at B on solid S_1.

Question 2.3 Determine the kinetic wrench of the beater S_3 in its motion with respect to R_0, at point O.

Question 2.4 Determine the dynamic wrench of the beater S_3 in its motion with respect to R_0, at point O.

Question 2.5 Determine the dynamic wrench of the plunger S_1 in its motion with respect to R_0, at point B.

Question 2.6 Establish the equation coming from the theorem of the dynamic sum, applied to S_1, along $\overrightarrow{z_{0,1}}$.

Question 2.7 Establish the equation coming from the theorem of the dynamic moment, applied to S_3, at O, along $\overrightarrow{x_{0,1,2,3}}$.