Mechanics of Systems – Test 1

Monday 27th November 2023 - 1h30 (10h15-11h45)

Authorised: Formula sheet (1.5 page + 1 sheet on classic joints) and Calculator

Indicative marking scheme: preliminary questions 2 marks; part A: 7 marks; part B: 11 marks

The preliminary questions make it possible to tackle the two independent parts A and B.

STUDY OF A CHILDREN'S DIGGER

An E.BECKMANN digger is studied, which has been entirely re-designed at INSA Lyon in order to propose this Mechanics test. .

Figure 1: Children's digger and Solid Edge re-design

The planar kinematical model in **figure 2** comprises **5** solids connected by **revolute joints** of axes collinear to $\overrightarrow{X_0}$. Note that 2 revolute joints are superimposed at point A: one linking **2** and **0** and another one between **1** and **0**.

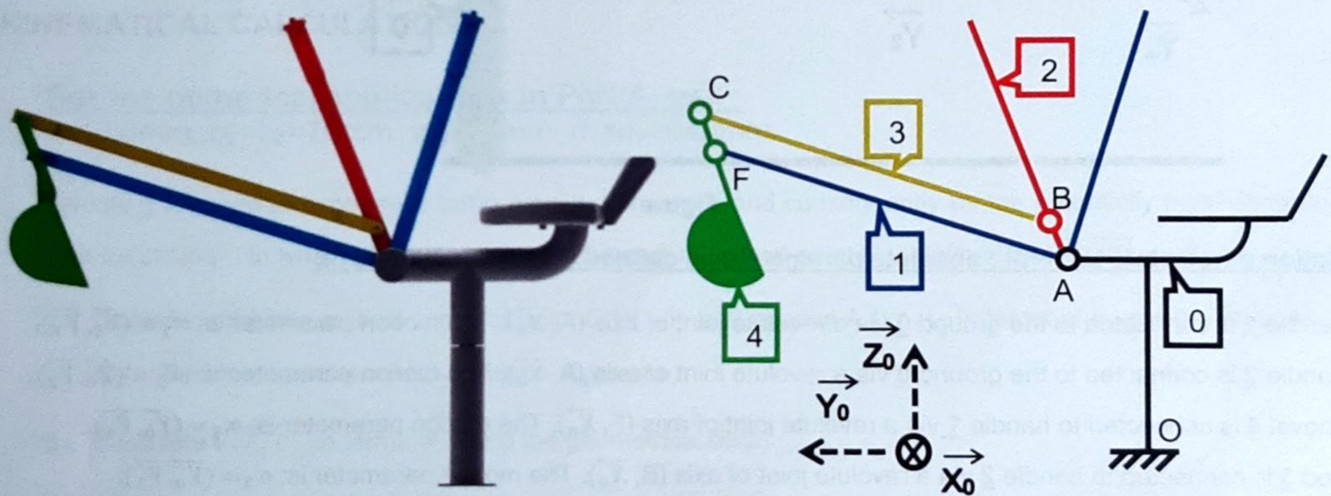


Figure 2: Kinematical model of a digger

To simplify the subsequent analyses, the rotation of the seat about the axis $(O,\overrightarrow{Z_0})$ is considered as blocked, so that the study can be limited to the plane $(O,\overrightarrow{Y_0},\overrightarrow{Z_0})$.

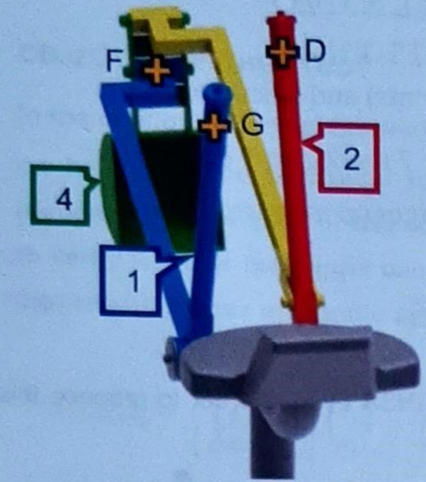


Figure 3

Operating the digger (figure 3):

The child holds the handles at points G and D.

By moving the left handle (in blue) labelled **1**, he can modify the altitude of shovel **4**.

By moving the right handle (in red) labelled **2**, he can tilt the shovel about point F in order to fill it in or empty it (with soil or sand).

Parameter definition

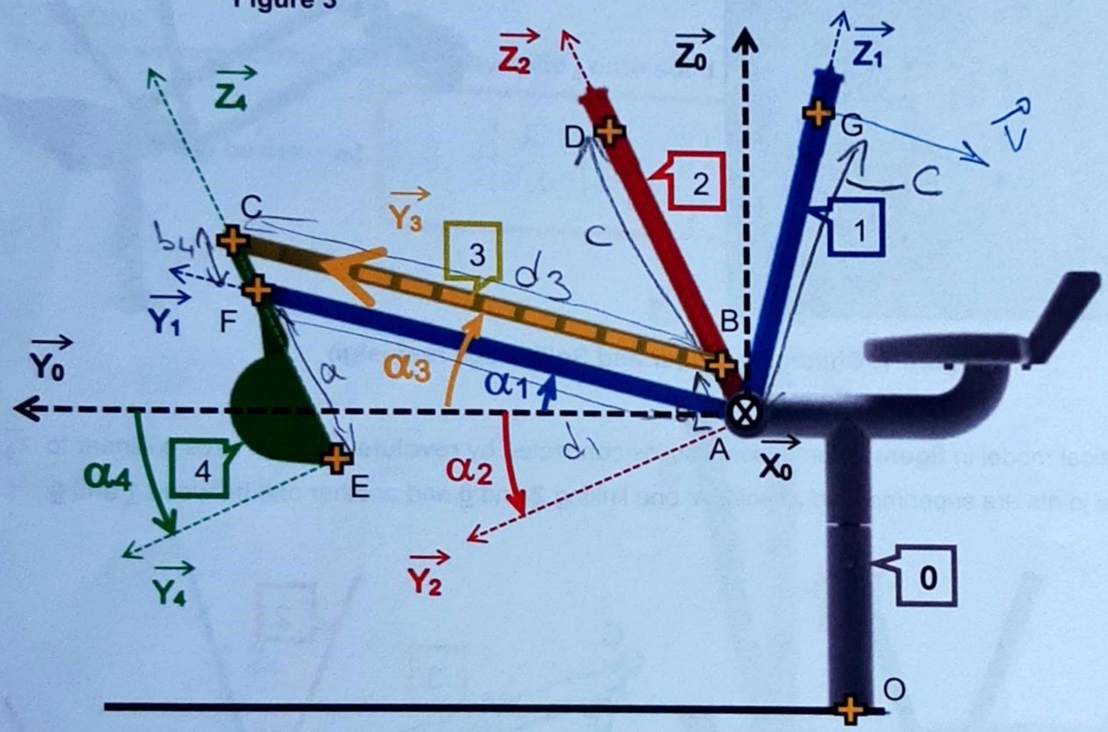


Figure 4

Motion parameters (careful : absolute parameters all defined with respect to R_0 !!):

Handle **1** is connected to the ground **0** via a revolute joint of axis (A, $\overrightarrow{X_0}$). The motion parameter is: $\alpha_1 = (\overrightarrow{Y_0}, \overrightarrow{Y_1})$.

Handle **2** is connected to the ground **0** via a revolute joint of axis (A, $\overrightarrow{X_0}$). The motion parameter is: $\alpha_2 = (\overrightarrow{Y_0}, \overrightarrow{Y_2})$.

Shovel **4** is connected to handle **1** via a revolute joint of axis (F, $\overrightarrow{X_0}$). The motion parameter is: $\alpha_4 = (\overrightarrow{Y_0}, \overrightarrow{Y_4})$.

Rod **3** is connected to handle **2** via a revolute joint of axis (B, $\overrightarrow{X_0}$). The motion parameter is: $\alpha_3 = (\overrightarrow{Y_0}, \overrightarrow{Y_3})$.

Moreover, shovel **4** is connected to rod **3** via a revolute joint of axis (C, $\overrightarrow{X_0}$). No parameter for this joint.

Geometrical data :

$\overrightarrow{AG} = c\overrightarrow{Z_1}$; $\overrightarrow{AD} = c\overrightarrow{Z_2}$; $\overrightarrow{AB} = b_2\overrightarrow{Z_2}$; $\overrightarrow{BC} = d_3\overrightarrow{Y_3}$; $\overrightarrow{FC} = b_4\overrightarrow{Z_4}$; $\overrightarrow{AF} = d_1\overrightarrow{Y_1}$; $\overrightarrow{EF} = a\overrightarrow{Z_4}$

HYPOTHESES

- The weights of all the solids are neglected compared with the other forces exerted on the digger parts.
- The problem is planar.
- All the joints are perfect (friction is neglected).

PRELIMINARY QUESTIONS

1- Based on the model description, draw the graph of links.

2- Draw the change of basis diagrams.

PART A : KINEMATICS OF A CHILDREN'S DIGGER

CONSTRAINT EQUATIONS - MOBILITY

3- Express the constraint condition(s) corresponding to the closure in the graph of links

4- Develop the scalar equations associated with the constraint condition(s). The results will be projected in the coordinate system R_0

5- What is the degree of mobility of the system? What would be the functional motion(s) associated with this degree of mobility?

6- (*Bonus optional question*) If the system is to be used so that point E of the shovel moves horizontally on the ground (see Figure 4), what would be the additional constraint equation (do not develop)? What could then be deduced about the use of the handles in this configuration?

KINEMATICAL CALCULATIONS

For the numerical applications in Part A, use :
$a=250mm$, $b_2=b_4=70mm$, $c=400mm$, $d_1=d_3=640mm$

Handle **1** is prone to experience large amplitude motions and consequently be the potentially most dangerous part for children. In what follows, the child's left-hand velocity at point G is defined as $\overrightarrow{V(G/0)} = v\overrightarrow{Y_1}$.

7- Determine the kinematical screw at point G $\{V_{1/0}\}_G$ and at point A $\{V_{1/0}\}_A$ in terms of v and the geometrical data. Deduce the relationship between $\dot{\alpha}_1$ and v.

8- Calculate $\overrightarrow{V(F/0)}$ in terms of v and the geometrical data.

9- Numerical application: $v = 0{,}5$ m/s, calculate $||\overrightarrow{V(F/0)}||$. For safety reasons, point F should move with respect to the ground with a speed below 1m/s. Is it the case in this example?

PART B : STATIC ANALYSIS OF A CHILDREN'S DIGGER

OBJECTIVE OF THE STUDY :

In the configuration represented in figure 5, the user wants to fill in the shovel with soil by exerting a force on handle 2. The horizontal force $\vec{F}$ exerted by the soil on shovel 4 can vary between 10 and 30N (depending if it is sand or wet soil). Considering the maximum force of 30 N, the objective in this section is to verify whether the digger can reasonably be manipulated by a 4-year old child, who cannot operate the digger with a force on handle above 20 N for more than one hour.

$$\{F_{soil/4}\}_E = \left\{ \begin{pmatrix} 0 \\ F \\ 0 \end{pmatrix}_{0,2,4} \middle| \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{0,2,4} \right\}_E , \{F_{\text{right hand}/2}\}_D = \left\{ \begin{pmatrix} 0 \\ F_d \\ 0 \end{pmatrix}_{0,2,4} \middle| \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{0,2,4} \right\}_D , \{F_{\text{left hand}/1}\}_G = \left\{ \begin{pmatrix} 0 \\ F_g \\ 0 \end{pmatrix}_{1,3} \middle| \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{1,3} \right\}_G$$

NOTATIONS

The force wrench at point P exerted by solid i onto solid j, expressed in the coordinate system

$\mathcal{B}_k(\vec{x}_k, \vec{y}_k, \vec{z}_k)$ will be denoted $\{F_{i/j}\}_P = \left\{ \begin{matrix} \vec{R}_{ij} \\ \vec{M}_{ij}(P) \end{matrix} \right\} = \left\{ \begin{pmatrix} X_{ij} \\ Y_{ij} \\ Z_{ij} \end{pmatrix}_k \middle| \begin{pmatrix} L_{ij} \\ M_{ij} \\ N_{ij} \end{pmatrix}_k \right\}_P$ with this notation $X_{ij} = -X_{ji}$

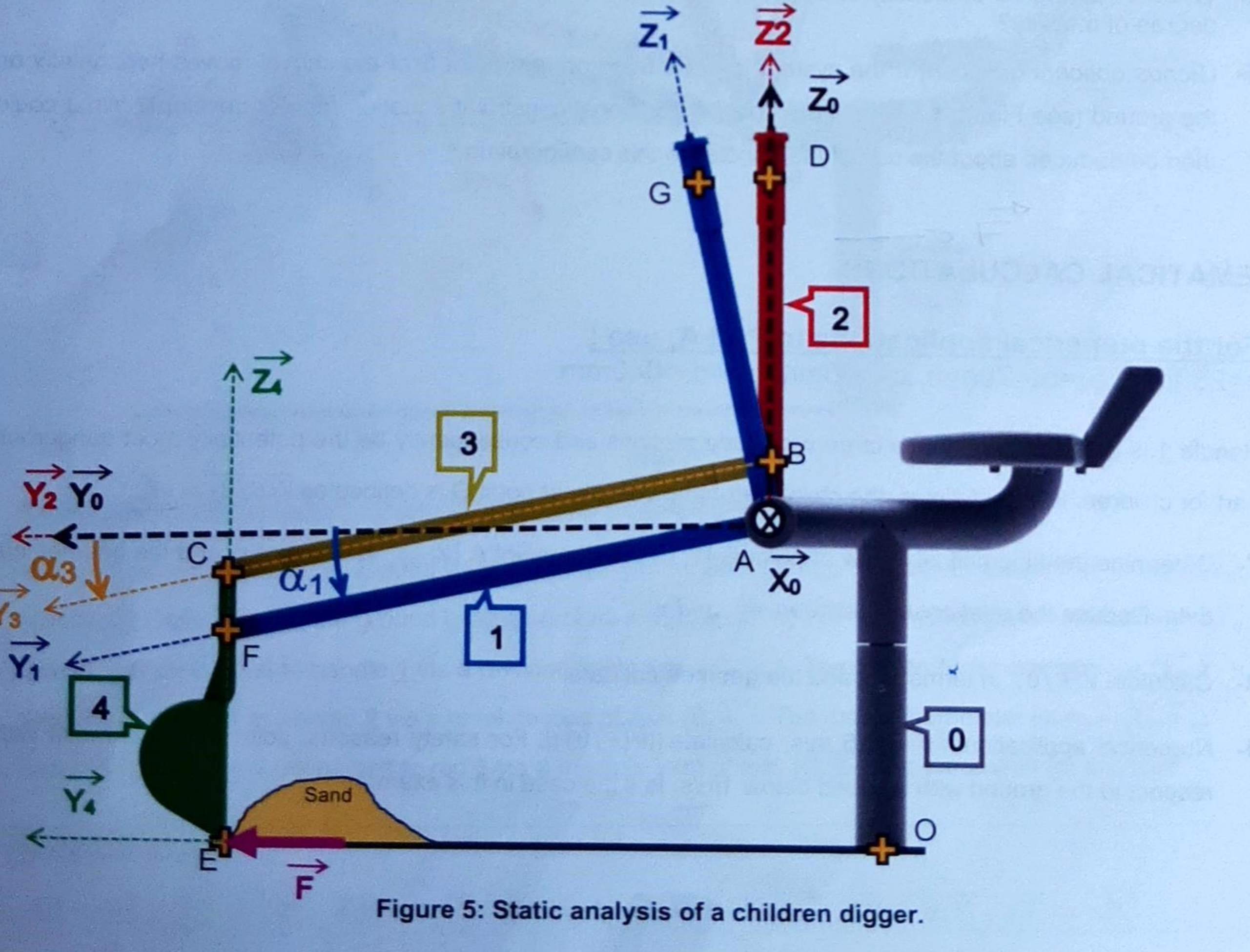


Figure 5: Static analysis of a children digger.

For the configuration of the static analysis (figure 5), the solids are in a particular position such that:

$\overrightarrow{Z_0} = \overrightarrow{Z_2} = \overrightarrow{Z_4}$ $\quad\quad$ $\overrightarrow{Z_1} = \overrightarrow{Z_3}$ $\quad\quad$ $\alpha_1 = \alpha_3 = -12°$ $\quad\quad$ $\alpha_2 = \alpha_4 = 0°$

moreover $\quad$ $b_2 = b_4$, $\quad$ $d_1 = d_3$

In this section, note that the change of basis diagrams defined in the preliminary questions are particular since $\mathcal{B}_0 = \mathcal{B}_2 = \mathcal{B}_4$ et $\mathcal{B}_1 = \mathcal{B}_3$.

10- Isolating **3**, prove that the component of $\vec{R}_{43}$ in the $\overrightarrow{Z_3}$ direction is nil and give the relationship between $\vec{R}_{34}$ and $\vec{R}_{23}$.

11- Isolating solid **4**, apply the static equilibrium equations at point F expressed in the coordinate system $\mathcal{B}_1$ $(\overrightarrow{x_1}, \overrightarrow{y_1}, \overrightarrow{z_1})$. Develop without solving the corresponding system of equations in terms of a, b_2, b_4, d_1, d_3 and the components of $\vec{R}_{34}$, *and* $\vec{R}_{14}$, force F and angles α_i with $i \in \{1, 2, 3, 4\}$.

12- Isolating solid **2**, apply the static equilibrium equations at point A. Develop without solving the corresponding system of equations.

13- Express force F_d in terms of force F

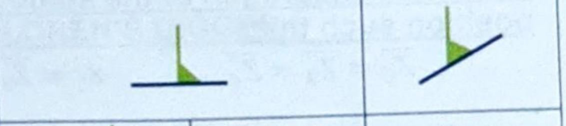
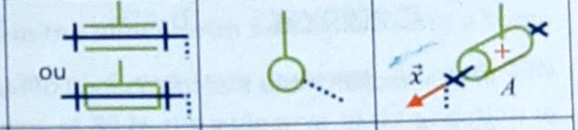
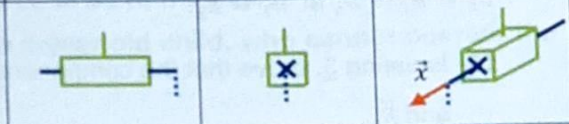
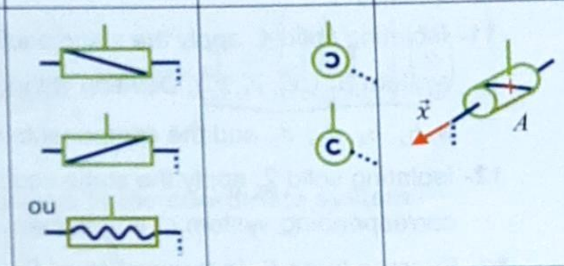
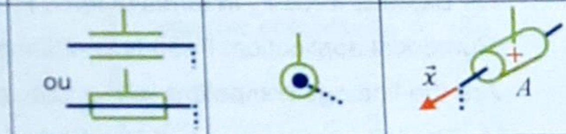
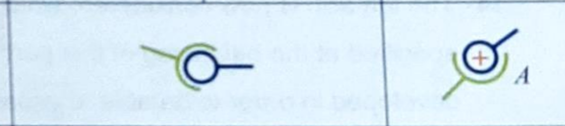
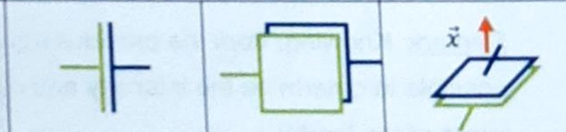
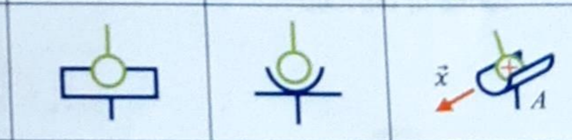
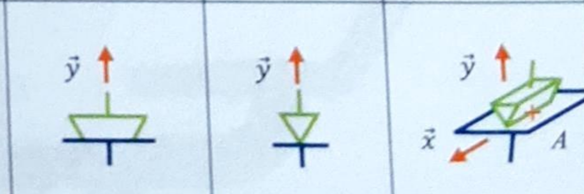
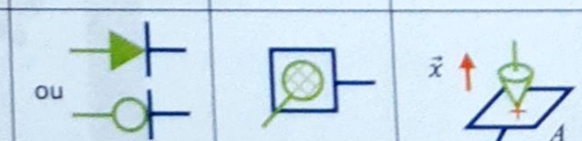
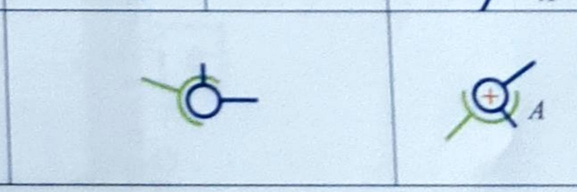
Numerical application: F=30 N, a=250mm, $b_2 = b_4 = 70$mm, c=400mm, $d_1 = d_3 = 640$mm.

Are the findings compatible with a force around 20N possibly exerted by a 4-year old child?

14- The left arm is now considered, which exerts on handle **1** a force of magnitude F_g whose wrench has been specified at the beginning of this part B. Which equation(s) derived from static equilibrium should be developed in order to be able to express F_g in terms of the coordinates of $\{F_{1/4}\}$ only?

Remark: Knowing, from the previous equations, that $\{F_{1/4}\}$ can be expressed in term of F, it would then be possible to determine the intensity and direction of the force that should be exerted by the child's left hand in terms of the load F.

Standard joints

joint	l_{ij}	m_{ij}	$\{\mathcal{V}\}$ - Kinematic wrench	$\{\mathcal{F}\}$ - Force wrench	Where ?	Plane representation (→ $\vec{x}$)	Plane representation (⊙ $\vec{x}$)	3D representation
Rigid	6	0	$\begin{Bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_A$	$\begin{Bmatrix} X & L \\ Y & M \\ Z & N \end{Bmatrix}_A$	Everywhere			
Revolute of axis $(A, \vec{x})$	5	1	$\begin{Bmatrix} \omega_x & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_A$	$\begin{Bmatrix} X & 0 \\ Y & M \\ Z & N \end{Bmatrix}_A$	On the line $(A, \vec{x})$			
Prismatic of axis $\vec{x}$	5	1	$\begin{Bmatrix} 0 & v_x \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_A$	$\begin{Bmatrix} 0 & L \\ Y & M \\ Z & N \end{Bmatrix}_A$	Everywhere			
Screw joint of axis $(A, \vec{x})$ and thread p	5	1	$\begin{Bmatrix} \omega_x & v_x \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_A$ $v_x = k\omega_x$	$\begin{Bmatrix} X & L \\ Y & M \\ Z & N \end{Bmatrix}_A$ $L = -kX$	On the line $(A, \vec{x})$			
			Translation and rotation are linked/dependant					
Cylindrical of axis $(A, \vec{x})$	4	2	$\begin{Bmatrix} \omega_x & v_x \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_A$	$\begin{Bmatrix} 0 & 0 \\ Y & M \\ Z & N \end{Bmatrix}_A$	On the line $(A, \vec{x})$			
Spherical of center A	3	3	$\begin{Bmatrix} \omega_x & 0 \\ \omega_y & 0 \\ \omega_z & 0 \end{Bmatrix}_A$	$\begin{Bmatrix} X & 0 \\ Y & 0 \\ Z & 0 \end{Bmatrix}_A$	At point A			
Planar of normal $\vec{x}$	3	3	$\begin{Bmatrix} \omega_x & 0 \\ 0 & v_y \\ 0 & v_z \end{Bmatrix}_A$	$\begin{Bmatrix} X & 0 \\ 0 & M \\ 0 & N \end{Bmatrix}_A$	Everywhere			
Spherical groove of center A and axis $(A, \vec{x})$	2	4	$\begin{Bmatrix} \omega_x & v_x \\ \omega_y & 0 \\ \omega_z & 0 \end{Bmatrix}_A$	$\begin{Bmatrix} 0 & 0 \\ Y & 0 \\ Z & 0 \end{Bmatrix}_A$	At point A			
Cylinder-plane joint of contact line $(A, \vec{x})$ and normal $\vec{y}$	2	4	$\begin{Bmatrix} \omega_x & v_x \\ \omega_y & 0 \\ 0 & v_z \end{Bmatrix}_A$	$\begin{Bmatrix} 0 & 0 \\ Y & 0 \\ 0 & N \end{Bmatrix}_A$	On the plane $(A, \vec{x}, \vec{y})$			
Point-plane of normal $(A, \vec{x})$	1	5	$\begin{Bmatrix} \omega_x & 0 \\ \omega_y & v_y \\ \omega_z & v_z \end{Bmatrix}_A$	$\begin{Bmatrix} X & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_A$	On the normal line $(A, \vec{x})$			
« Spherical joint with an ergot » no true equivalent to this joint in english center A and axis $\vec{x}$	4	2	$\begin{Bmatrix} 0 & 0 \\ \omega_y & 0 \\ \omega_z & 0 \end{Bmatrix}_A$	$\begin{Bmatrix} X & L \\ Y & 0 \\ Z & 0 \end{Bmatrix}_A$	At point A			

↑ Degree of constraint (l_{ij})

↑ Degree of mobility (m_{ij})